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THREE-DIMENSIONAL RAY TRACE
COMPUTER PROGRAM FOR ELECTROMAGNETIC
WAVE PROPAGATION STUDIES



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THREE-DIMENSIONAL RAY TRACE COMPUTER PROGRAM
FOR ELECTROMAGNETIC WAVE PROPAGATION STUDIES,

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ABSTRACT

A computer program is described for use on the IBM-704/7090 electronic data processing machine or any large computer accepting FORTRAN. The necessary modifications for use on these two computers are simple and self-evident. The computer program permits the computation of detailed ray patterns portraying the spatial distribution of rays emitted from a transmitter whose geographic coordinates with respect to the center of the earth are known. This program deals with the solution of the differential equations, given by Hamiltonian optics, for ray paths in non-isotropic, three-dimensionally nonhomogeneous media whose complex phase refractive index is given by the Appleton-Hartree formula.

This report is to be considered as a first attempt in presenting an account of the current status of the development of this program, which has yielded many useful results.* Presented also are sample calculations as well as some results that have been obtained by using this program.

* see p. 111.

TABLE OF CONTENTS

	Page
ABSTRACT	iii
LIST OF ILLUSTRATIONS	vii
LIST OF TABLES	ix
SECTION	
I INTRODUCTION	1
II COMPUTATIONAL PROCEDURE	3
A. Ray Trace Equations	3
B. List of Useful Formulae for Ray Tracing	11
C. Coordinate Transformation	17
D. Model Ionosphere	31
E. Model of Earth's Magnetic Field	33
F. Model of Atmospheric Collision Frequency	35
G. Computational Results	36
III COMPUTER PROGRAM FOR THREE- DIMENSIONAL RAY-TRACING	41
A. Main Program RAY TRACE	45
B. Function SLANTR	65
C. Function QATAN	69
D. Function ARCCOS	73
E. Subroutine COORD	77
F. Subroutine DAUX	82

SECTION	Page
III	86
G. Subroutines INT and INTM	86
H. Subroutine RINDEX	112
I. Subroutine ELECTX	129
J. Subroutine BIGR	133
K. Subroutine MAGY	137
L. Subroutine COLFRZ	141
M. Subroutine RCOORD	145
N. Subroutine OUTONE	150
O. Subroutine OUTPUT	154
P. INPUT-OUTPUT	159
ACKNOWLEDGEMENTS	167
REFERENCES	169

LIST OF ILLUSTRATIONS

Figure	Title	Page
1a	Geometry of Coordinate Transformation	18
1b	Geometry of Coordinate Transformation	19
1c	Geometry of Coordinate Transformation	21
1d	Geometry of Coordinate Transformation	22
2	Geometry for Starting Point and Geomagnetic Coordinate System	28
3	Geometry for Refraction by Spherically Stratified Region	37
4	Radar Propagation Paths Through Spherically Ionized Region	43
5	Elevation and Azimuth Errors for Propagation through a Spherical Model - $f = 1k \text{ Mc}$	44
6	Block Diagram of Three-Dimensional Ray-Tracing Program	44a

LIST OF TABLES

Table	Title	Page
1	Comparison of Total Ray Bending Angle	38
2	Nomenclature describing the V and W Vectors	48
3	Nomenclature Describing the R Vector	113
4	Input Data for a Spherical Ionosphere	160
5	Output Data on Tape 6 for Debugging Purposes	161
6	Output Data From Tape 10	163

SECTION I INTRODUCTION

What is customarily referred to as reflection in electromagnetic wave propagation, is actually the result of an integrated effect of a phenomenon of refraction (i. e., the gradual changing of the direction of the electromagnetic energy transport vector). Insofar as the phenomenon of refraction is concerned it is well established that the spatial gradient of electron density plays a crucial role in controlling the propagation behavior of an electromagnetic wave. Although this is well known, propagation studies so frequently incorporate a curious mixture of simple refraction phenomena (in the use of Snell's law) and of mirror-like reflection. Hence, in these studies they omit an accounting of these spatial gradients of electron density as well as their variation. As generally known, these oversimplified studies derive from Snell's law the condition for reflection, that being, that at the spatial point of reflection, the electron density must attain the value of $1.24 \times 10^4 f^2 \cos^2 I$ electron/cc, where f is the propagation frequency in Mc/sec and I is the angle of incidence of the ray upon the first layer of the assumed stratified ionosphere. Such a simplified approach is necessary because of the difficult task that leads to a numerical solution of a propagation problem which incorporates these electron density gradients in its solution.

With increasing use of transmitters in satellites, as well as, for the understanding of the behavior of wave propagation under severely abnormal atmospheric conditions, it becomes important to take a realistic account of the spatial gradient of the electron density in electromagnetic wave propagation studies. Only in this manner will it become possible to usefully utilize the new satellite propagation techniques in studies designed toward the understanding of the atmospheric ionization-deionization phenomena and through this the detailed structure of the ionosphere.

Within the last ten years some effort has been made in constructing analogue computers for the study of ray propagation which accounted for spatial electron density gradients (as for example D. F. Hartree, et al., Manchester, England; M. S. Wong, AFCRL, Bedford, Mass.).

Some of these analogue computers were and still are limited to spatial electron density gradients in a particular direction thus forcing the propagation problem into a two-dimensional plane, or to the study of the behavior of refraction on wave propagation. These constraints are built into the analogue computer and are not easily changed.

One approach that avoids these constraints is the use of a large electronic data processing system where the ordered logical flow controlling any calculation, is easily varied. Combining such a programmed computer with Hamiltonian optics, which give the desired ray tracing equations for a nonhomogeneous, non-isotropic medium, and the Appleton-Hartree formula for the complex refractive index, permits in addition to three dimensional ray tracing, the simultaneous study of numerous other variables of the propagation problem. Such an approach to the ray trace propagation problem is presented in its present state of development. A great deal of improvement in some of the routines is possible. As a result, the writer would like to encourage correspondence concerning these matters. In addition, it should be stated, that a computer program for solving the three-dimensional ray trace problem has also been written for the Ferranti Mercury Computer at Manchester University by C. B. Haselgrove and J. Haselgrove.²

SECTION II

COMPUTATIONAL PROCEDURE

A. RAY TRACE EQUATIONS

When electromagnetic waves are propagated through a medium whose dielectric constant or index of refraction is a varying function of the path, the waves undergo a change in direction, or refractive bending, and a retardation in the velocity of propagation. The spatial relationship expressing this angular bending of a ray of an electromagnetic wave can be determined by basing the theory of rays and waves on a variational principle (Fermat's) in space. By a ray is meant the path travelled by the transport vector of electromagnetic wave energy between the transmitter and an associated electromagnetic field-intensity point in space. This Hamilton Theory starts from the variational principle $\delta \int ds = 0$, where μ is a medium function or index of refraction, depending on position and direction. From this principle the theory constructs the properties of systems of rays and the waves associated with them (i.e., extremals and transversals, in the language of the calculus of variations). Because of the stationarity in time, the theory may be regarded as a statical one, the rays being fixed curves in space and the waves fixed surfaces. Neither wave-length nor frequency is involved. Likewise the waves form a continuous set of surfaces, not distinguished as crests and troughs. This theory, whether in the form preferred by Hamilton or otherwise, has been the subject of many books under the general title "Geometric Optics".

Thus, applying Hamiltonian optics leads to the general Hamilton's Equations¹ for ray paths of electromagnetic waves in a three-dimensional non-isotropic nonhomogeneous medium. From them Haselgrove² has derived the following set of equations for ray paths in a spherical coordinate system in a format suitable for numerical integration on high speed computers:

$$\frac{dr}{d\tau} = \frac{1}{\mu^2} \left(\sigma_r - \mu \frac{\partial \mu}{\partial \sigma_r} \right) \quad (1)$$

$$\frac{d\theta}{d\tau} = \frac{1}{r\mu^2} \left(\sigma_\theta - \mu \frac{\partial \mu}{\partial \sigma_\theta} \right) \quad (2)$$

$$\frac{d\varphi}{d\tau} = \frac{1}{\mu^2 r \sin \theta} \left(\sigma_\varphi - \mu \frac{\partial \mu}{\partial \sigma_\varphi} \right) \quad (3)$$

$$\frac{d\sigma_r}{d\tau} = \frac{1}{\mu} \frac{\partial \mu}{\partial r} + \sigma_\theta \frac{d\theta}{d\tau} + \sin \theta \sigma_\varphi \frac{d\varphi}{d\tau} \quad (4)$$

$$\frac{d\sigma_\theta}{d\tau} = \frac{1}{r} \left[\frac{1}{\mu} \frac{\partial \mu}{\partial \theta} - \sigma_\theta \frac{dr}{d\tau} + r \cos \theta \sigma_\varphi \frac{d\varphi}{d\tau} \right] \quad (5)$$

$$\frac{d\sigma_\varphi}{d\tau} = \frac{1}{r \sin \theta} \left[\frac{1}{\mu} \frac{\partial \mu}{\partial \varphi} - \sin \theta \sigma_\varphi \frac{dr}{d\tau} - r \cos \theta \sigma_\theta \frac{d\theta}{d\tau} \right] \quad (6)$$

In these equations r , θ , and φ are the spatial coordinates of a spherical system; μ is the arbitrary index of refraction; $\vec{\sigma}$ is a vector directed normal to the phase fronts of the ray of magnitude μ with σ_r , σ_θ , and σ_φ its respective components in the r , θ , and φ directions; τ is the time of phase travel along the ray path (i. e., $f\Delta\tau/c$ = the number of wavelengths in the medium along the ray path, where f is the electromagnetic wave frequency and c the velocity of light in vacuum).

It is noteworthy that the partial derivatives of μ appear explicitly in Equations 1 to 6, in accordance with the fact that the gradients of μ play crucial roles in determining the spatial ray paths.

This closed set of first-order partial differential equations which will describe the propagation behavior of an electromagnetic wave

under geometric optics conditions, can be integrated simultaneously by a point-by-point numerical process if expressions can be developed for the necessary derivatives of the phase refractive index μ . The quantity μ and its derivatives are obtained by using the Appleton-Hartree formula³ as the definition of the complex phase refractive index M .

The derivatives are derived under the conditions of ray optics, that is, that the imaginary part of M^2 is very much smaller than the real part. As an aid for computer use and comparison with published works of others^{2,4}, the Appleton-Hartree formula is written as:

$$M^2 = (\mu - j\kappa)^2 = 1 - \frac{2X(1 - X - jZ)}{D} \quad (7)$$

$$D = 2(1 - X - jZ)(1 - jZ) - Y^2 \sin^2 \psi + S \quad (8)$$

$$S = \pm \left[(Y \sin \psi)^4 + 4Y^2(1 - X - jZ)^2 \cos^2 \psi \right]^{1/2} \quad (9)$$

where

M = the complex phase refractive index

X = a scalar quantity representing the normalized electron density

$$\frac{4\pi Ne^2}{m\omega^2} = \frac{\omega_p^2}{\omega^2}$$

ω_p = plasma frequency at the spatial point

ω = angular wave frequency = $2\pi f$

m, e = mass and charge of an electron

N = electron density at a spatial point

Y = normalized magnitude of the earth's magnetic field
vector \vec{Y} =

$$\frac{eH}{mc\omega} = \frac{\vec{\omega}_c}{\omega}$$

ω_c = magnitude of the gyromagnetic frequency of the electron
about the earth's magnetic field

Z = normalized collision frequency = (v/ω)

v = collision frequency at a spatial point in collisions per
second

ψ = angle defined by the inner product of the vectors $\vec{\delta}$ and
 \vec{Y} =

$$\cos^{-1} \left[\frac{\sigma_r Y_r + \sigma_\theta Y_\theta + \sigma_\phi Y_\phi}{(\mu Y)} \right]$$

κ = $\frac{ck}{\omega}$ = imaginary part of the complex phase refractive
index

c = velocity of light in vacuum

k = absorption coefficient of the wave per unit length of
path (it is proportional to the conductivity of the medium)

It is noted that there are two possible values for the complex index
of refraction M corresponding to the plus and minus sign on S
which represent two different modes of ionospheric propagation.
These are commonly called "ordinary" and "extraordinary" modes
for the plus and minus sign respectively. Also, the Appleton-Hartree
formula (Equations 7 to 9) is notable in that it presents μ , which
actually is a spatial function of six variables, in the form containing
purely algebraic operations on factors or terms each of which is a
function of at most three variables, that is, either of r , θ , Ψ or σ_r ,
 σ_θ , σ_ϕ . This reduces the representation of μ to a numerical
problem, easily solvable to current computer techniques.

If i represents any one of the spatial spherical coordinates r , θ , and Ψ , then the partial derivatives of μ with respect to the components of the wave normal, σ , can be easily shown to be

$$\frac{\partial \mu}{\partial \sigma_i} = \frac{\partial \mu}{\partial \Psi} \frac{\partial \Psi}{\partial \sigma_i} = \frac{\partial \mu}{\partial \Psi} \left(\frac{\sigma_i Y \cos \Psi - \sigma Y_i}{\sigma^2 Y \sin \Psi} \right) \quad (10)$$

This useful transformation also enjoys the following property:

$$\text{When } \Psi \rightarrow 0, \frac{\partial \mu}{\partial \Psi} \rightarrow 0, \frac{\partial \Psi}{\partial \sigma_i} \rightarrow \infty \quad \text{but} \quad \frac{\partial \mu}{\partial \sigma_i} \rightarrow 0 \quad (11)$$

To evaluate $\partial \mu / \partial \sigma_i$; the necessary partial derivative is:

$$\begin{aligned} \frac{\partial \mu}{\partial \Psi} &= \operatorname{Re} \frac{\partial M}{\partial \Psi} = \operatorname{Re} \left\{ \frac{(M^2 - 1)(Y^2 \sin \Psi \cos \Psi)}{MD} \left[1 - \frac{1}{S} [(Y \sin \Psi)^2 - 2(1 - X - jZ)^2] \right] \right\} \\ \therefore \frac{\partial \mu}{\partial \Psi} &= \left\{ -Y^2 \sin \Psi \cos \Psi [a_0(a_2 a_5 - b_2 b_5) - b_0(a_2 b_5 + b_2 a_5)] \right\} \end{aligned} \quad (12)$$

where a_0 , b_0 and all following a_j , b_j are defined in the List of Useful Formulae. The partial derivatives of the real part of the phase refractive index with respect to the spatial coordinates ($i = r$, θ , or Ψ) are similarly obtained by use of the relationship

$$\frac{\partial \mu}{\partial i} = \frac{\partial \mu}{\partial X} \frac{\partial X}{\partial i} + \frac{\partial \mu}{\partial Y} \frac{\partial Y}{\partial i} + \frac{\partial \mu}{\partial Z} \frac{\partial Z}{\partial i} + \frac{\partial \mu}{\partial \Psi} \frac{\partial \Psi}{\partial i} \quad (13)$$

where

$$\frac{\partial \mu}{\partial X} = \operatorname{Re} \frac{\partial M}{\partial X} = \operatorname{Re} \left\{ \frac{1}{MD} \left[2X - 1 + jZ + (M^2 - 1)(1-jZ + \frac{2Y^2(1-X-jZ)\cos^2\psi}{S}) \right] \right\}$$

$$\therefore \frac{\partial \mu}{\partial X} = \left\{ a_0 \left[(2X-1) - (a_4 a_5 - b_4 b_5) \right] + b_0 \left[Z + a_5 b_4 + b_5 a_4 \right] \right\} \quad (14)$$

$$\frac{\partial \mu}{\partial Y} = \operatorname{Re} \frac{\partial M}{\partial Y} = \operatorname{Re} \left\{ \frac{(M^2 - 1)}{MD} Y \left[(\sin \psi)^2 - \frac{1}{S} [Y^2 \sin^4 \psi + 2(1-X-jZ)^2 \cos^2 \psi] \right] \right\}$$

$$\therefore \frac{\partial \mu}{\partial Y} = Y \left\{ (a_0 a_5 - b_0 b_5) \left[a_6 - (\sin \psi)^2 \right] - b_6 (a_0 b_5 + b_0 a_5) \right\} \quad (15)$$

$$\frac{\partial \mu}{\partial Z} = \operatorname{Re} \frac{\partial M}{\partial Z} = - \operatorname{Im} \left\{ \frac{1}{MD} \left[X + (M^2 - 1)(2 - X - 2jZ + \frac{2Y^2(1-X-jZ)}{S} \cos^2 \psi) \right] \right\}$$

$$\frac{\partial \mu}{\partial Z} = \left[b_0 (X - a_5 a_7 + b_5 b_7) - a_0 (b_5 a_7 + a_5 b_7) \right] \quad (16)$$

and where Re and Im represent, respectively, the real and imaginary part of the complex expression. The partial derivative of the angle ψ (which is the angle defined by the inner product of the normalized geomagnetic field vector \vec{H} and the wave normal vector $\vec{\delta}$), with respect to the spatial coordinates r , θ , and ϕ , measures the change in spatial direction of the earth's magnetic field since the calculation is made holding $\vec{\delta}$ constant. The partial derivatives $\partial X/\partial i$, $\partial Y/\partial i$ and $\partial Z/\partial i$ are obtainable from the analytical expressions for the spatial variation of the electron density, geomagnetic field and collision frequency. Examples of these will be considered later.

In addition to Equations 1 to 6, which define the spatial ray path, it is usually desirable to calculate the optical path length s , the time of travel T , as well as, the one way absorption A , of the energy of an electromagnetic pulse. The equation describing the differential optical path is given by

$$\frac{ds}{dt} = \frac{1}{\mu^2} \left[\mu^2 + \left(\frac{\partial \mu}{\partial V} \right)^2 \right]^{1/2} \quad (17)$$

In determining the time of travel T , a distinction must be made between two electromagnetic wave velocities. The phase velocity, $v_p = c/\mu$, is defined as the spatial velocity with which a point of constant phase moves. Group velocity, $v_g = d\omega/d(\omega/v_p)$, is the spatial velocity of electromagnetic energy travel; put into other words, it is the velocity of a "Maxwell Demon" who remains at the same point on the envelope of the advancing wave. From these two definitions it can be easily shown that the time (in seconds) of energy pulse travel can be written as:

$$\frac{dT}{dt} = \frac{1}{c} \left[1 + \frac{\omega}{\mu} \frac{\partial \mu}{\partial \omega} \right]; \quad (18)$$

where

$$\begin{aligned} \frac{\partial \mu}{\partial \omega} &= \operatorname{Re} \frac{\partial M}{\partial \omega} = \\ &- \operatorname{Re} \left\{ \frac{1}{MD\omega} \left[X(2X + jZ) + (M^2 - 1) \left[2 - 2jZ - jZX + \frac{2(Y \cos \psi)^2}{S} (1 - X - jZ)(1 + X) \right] \right] \right\} \\ \therefore \frac{\partial \mu}{\partial \omega} &= - \frac{1}{\omega} \left[a_o (2X^2 - a_5 a_8 + b_5 b_8) + b_o (XZ + b_5 a_8 + b_8 a_5) \right] \quad (19) \end{aligned}$$

The one-way absorption, A (in nepers), suffered by the energy of an electromagnetic pulse is determined by

$$\frac{dA}{d\tau} = - \frac{\omega k}{c \mu} A = - \frac{k}{\mu} A \quad (20)$$

where k (which is proportional to the spatial conductivity) is the absorption of the wave per unit length of path.

The solution of this set of first order partial differential equations will describe the propagation characteristics of an electromagnetic wave in a heterogeneous anisotropic medium.

B. LIST OF USEFUL FORMULAE FOR RAY TRACING

As an aid for the computer solution of these differential equations the following list of formulae are found to be very useful. As before, Re and Im respectively represent the real and imaginary parts of the complex quantity.

$$\text{Re}S = S_1 = R_S \cos \theta_S \quad (21)$$

$$\text{Im}S = S_2 = R_S \sin \theta_S \quad (22)$$

$$R_S = \left\{ \left[(Y \sin \psi)^4 + (2Y \cos \psi)^2 [(1 - X)^2 - Z^2] \right]^2 + \left[(2Y \cos \psi)^2 [2(1 - X)Z] \right]^2 \right\}^{1/4} \quad (23)$$

$$\theta_S = \frac{1}{2} \tan^{-1} \left\{ \frac{(2Y \cos \psi)^2 [2(X - 1)Z]}{(Y \sin \psi)^4 + (2Y \cos \psi)^2 [(1 - X)^2 - Z^2]} \right\} \quad (24)$$

$$\text{Re}D = d_1 = \left\{ 2[(1 - X)^2 - Z^2] - (Y \sin \psi)^2 + S_1 \right\} \quad (25a)$$

$$\text{Im}D = d_2 = [S_2 - 2Z(2 - X)] \quad (25b)$$

$$\text{Re}M = m_1 = \mu = R_m \cos \theta_m \quad (26)$$

$$\text{Im}M = m_2 = -\kappa = R_m \sin \theta_m \quad (27)$$

$$R_m \left\{ \left(1 - \frac{2X[(1-X)d_1 - Zd_2]}{d_1^2 + d_2^2} \right)^2 + \left(\frac{2X[Zd_1 + (1-X)d_2]}{d_1^2 + d_2^2} \right)^2 \right\}^{1/4} \quad (28)$$

$$\theta_m = \frac{1}{2} \tan^{-1} \left\{ \frac{2X[Zd_1 + (1-X)d_2]}{d_1^2 + d_2^2 - 2X[(1-X)d_1 - Zd_2]} \right\} \quad (29)$$

$$a_o = \frac{(m_1 d_1 - m_2 d_2)}{(m_1 d_1 - m_2 d_2)^2 + (m_1 d_2 + m_2 d_1)^2} \quad (30)$$

$$b_o = \frac{(m_1 d_2 + m_2 d_1)}{(m_1 d_1 - m_2 d_2)^2 + (m_1 d_2 + m_2 d_1)^2} \quad (31)$$

$$a_1 = \left\{ 2 \left[(1-X)^2 - Z^2 \right] - (Y \sin \psi)^2 \right\} \quad (32)$$

$$b_1 = 4Z(1-X) \quad (33)$$

$$a_2 = \left[1 + \frac{(a_1 s_1 - b_1 s_2)}{s_1^2 + s_2^2} \right] \quad (34)$$

$$b_2 = \left[\frac{s_1 b_1 + a_1 s_2}{s_1^2 + s_2^2} \right] \quad (35)$$

$$a_4 = \left\{ 1 + \frac{2(Y \cos \psi)^2}{s_1^2 + s_2^2} [s_1(1 - X) - Zs_2] \right\} \quad (36)$$

$$b_4 = \left\{ Z + \frac{2(Y \cos \psi)^2}{s_1^2 + s_2^2} [s_2(1 - X) + Zs_1] \right\} \quad (37)$$

$$a_5 = \frac{2X[(1 - X)d_1 - Zd_2]}{d_1^2 + d_2^2} = (1 + m_2^2 - m_1^2) \quad (38)$$

$$b_5 = \frac{2X[Zd_1 + (1 - X)d_2]}{d_1^2 + d_2^2} = 2m_2 \quad (39)$$

$$a_6 = \frac{s_1 \left\{ (Y \sin^2 \psi)^2 + 2 \cos^2 \psi [(1 - X)^2 - Z^2] \right\} - s_2 [(2 \cos \psi)^2 Z(1 - X)]}{s_1^2 + s_2^2} \quad (40)$$

$$b_6 = \frac{s_2 \left\{ (Y \sin^2 \psi)^2 + 2 \cos^2 \psi [(1 - X)^2 - Z^2] \right\} + s_1 [(2 \cos \psi)^2 Z(1 - X)]}{s_1^2 + s_2^2} \quad (41)$$

$$a_7 = \left\{ (2 - X) + \frac{2(Y \cos \psi)^2}{S_1^2 + S_2^2} [S_1(1 - X) - S_2 Z] \right\} \quad (42)$$

$$b_7 = \left\{ 2Z + \frac{2(Y \cos \psi)^2}{S_1^2 + S_2^2} [S_1 Z + S_2(1 - X)] \right\} \quad (43)$$

$$a_8 = 2 \left\{ 1 + \frac{(Y \cos \psi)^2}{S_1^2 + S_2^2} [(1 - X^2)S_1 - S_2 Z(1 + X)] \right\} \quad (44)$$

$$b_8 = \left\{ Z(2 + X) + \frac{2(Y \cos \psi)^2}{S_1^2 + S_2^2} [S_1 Z(1 + X) + S_2(1 - X^2)] \right\} \quad (45)$$

Nomenclature Used in Ray Trace Equations

r, θ, ψ	spatial coordinates of a spherical system
$\vec{\sigma}$	wave normal or refractive index vector
$\sigma_r, \sigma_\theta, \sigma_\phi$	vector components ($\vec{\sigma}$)
τ	time of phase travel (in units of length)
μ	real part of complex phase refractive index
f	electromagnetic wave frequency
c	velocity of light in vacuum
m, e	mass and charge of an electron
Re	real part of the complex expression
Im	imaginary part of the complex expression
M	complex phase refractive index
X	scalar quantity representing normalized electron density
ω_p	plasma frequency at the spatial point
ω	angular wave frequency
N	electron density at spatial point
Y	normalized magnitude of the earth's magnetic field vector \vec{Y}
ω_c	magnitude of the gyromagnetic frequency of the electron about the earth's magnetic field
Z	normalized collision frequency (v/ω)
v	collision frequency at spatial point in collisions/sec.
ψ	angle defined by inner product of vectors $\vec{\sigma}$ and \vec{Y}

κ	imaginary part of the complex phase refractive index
k	absorption coefficient of the wave per unit length of path (proportional to the conductivity of the medium)
i	represents each of the spatial spherical coordinates r, θ, φ
T	time of travel (seconds)
s	optical path length (km)
A	one-way absorption (nepers)
v_p	spatial velocity with which a point of constant phase moves
v_g	group velocity - spatial velocity of electromagnetic energy travel

C. COORDINATE TRANSFORMATION

If one takes three orthogonal planes intersecting at a point, one knows that the position of any point S in space is uniquely determined by the three perpendiculars from S on these planes, each with its proper sign. However, the problem of selecting the most useful orientation of such an orthogonal system is difficult since the usefulness of a coordinate system partially depends on the problem definition and the application of its solution. This ray trace program is designed as a sub-set of a much larger computer programming effort⁷ where the earth's geomagnetic field plays an important part. To minimize the number of computer transformations in the design of the over-all program, an earth centered spherical coordinate system (r, θ, φ) was chosen, whose z-component is coincident with the magnetic dipole axis.

This selection permits the application of the computer program to a great many studies of ray path problems because it accounts for the earth curvature and accepts for solution any electromagnetic radiating source whose transmitter location specifications of elevation, E, and azimuth, A, angles, as well as, geographic latitude, \varPhi_R , geographic longitude, λ_R , and position with respect to the surface of the earth are known. Because the usefulness of this computer program can be extended by modification to other coordinate systems, as for example, an earth centered geographic system, or a radar coordinate system, the necessary coordinate transformations from the radar to the earth centered geomagnetic coordinate system will be described in detail.

For the discussion of this coordinate transformation, it is assumed that the electromagnetic wave transmitter is earth bound (i. e., fixed to the surface of the earth) at a geographic latitude \varPhi_R and a geographic longitude λ_R . It is assumed that the radar is positioned so that the transmitting direction is described by the elevation angle, E, with respect to the tangent plane to the earth surface at the radar location, and an azimuth angle, A, measured from the radar coordinate that is tangent to a great circle passing through the north geographic pole. The azimuth angle is plus when measured counter-clockwise from the coordinate axis, ζ , whose positive direction points in the direction of geographic north. It is further assumed that S is a spatial point on the non-deviated portion of the ray a distance, R, from the electromagnetic wave transmitter. This is the spatial starting

point at which the numerical methods necessary for solution of the differential equations, must be initialized.

A transformation is required from the spherical radar coordinate system to the magnetic coordinate system whose origin is at the center of the earth.

The necessary matrices which are required to transform R, A, E coordinates to r, θ, ϕ coordinates can be arrived at by a series of simple matrix transformations.

1. Let ϵ, η, ζ be a set of orthogonal coordinates with origin on the surface of the earth. Let ϵ -axis be perpendicular to the earth's surface while ζ is directed (geographically) northward and η to the east. As stated above R is the slant range; E is the elevation angle; and A is the azimuth angle. Hence going from R, E, A to ϵ, η, ζ

$$\begin{pmatrix} \epsilon \\ \eta \\ \zeta \end{pmatrix} = \begin{pmatrix} R \sin E \\ R \cos E \sin A \\ R \cos E \cos A \end{pmatrix} \quad (46)$$

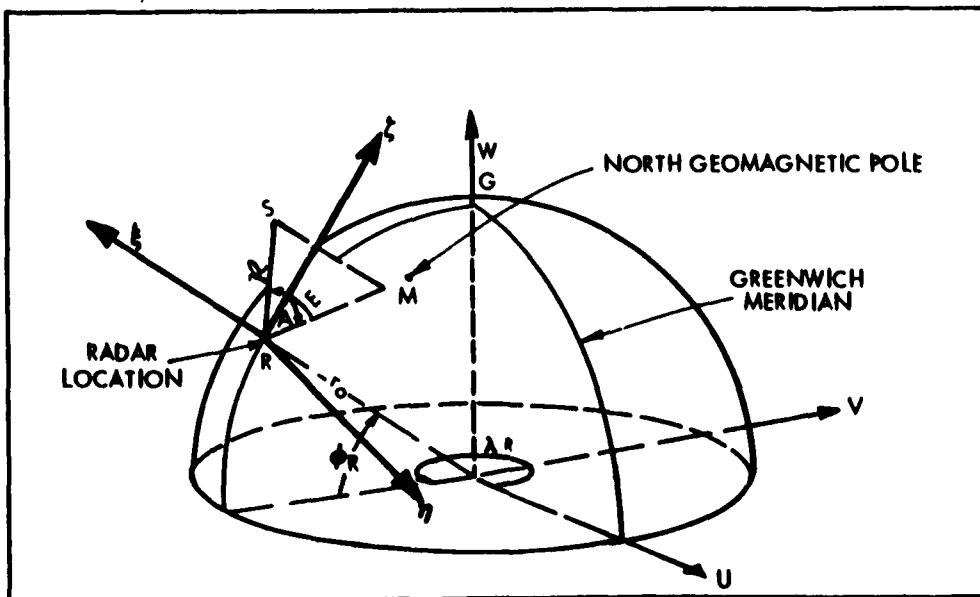


Figure 1a. Geometry of Coordinate Transformation

2. Let x_1, y_1, z_1 equal an orthogonal coordinate system with origin on the earth's axis of rotation. The x_1 -axis is in the latitude plane of the radar site and passes through the radar site. The z_1 -axis is coincident with the north geographic coordinate w. A translation and rotation is required in going from ϵ, η, ζ to x_1, y_1, z_1 .

$$\begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} = \begin{pmatrix} \cos\varphi_R & 0 & -\sin\varphi_R \\ 0 & 1 & 0 \\ \sin\varphi_R & 0 & \cos\varphi_R \end{pmatrix} \begin{pmatrix} \epsilon \\ \eta \\ \zeta \end{pmatrix} + \begin{pmatrix} r_o \cos\varphi_R \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} = (b_{ij}) \begin{pmatrix} \epsilon \\ \eta \\ \zeta \end{pmatrix} + (b_i) \quad (47)$$

where r_o equals the earth's radius.

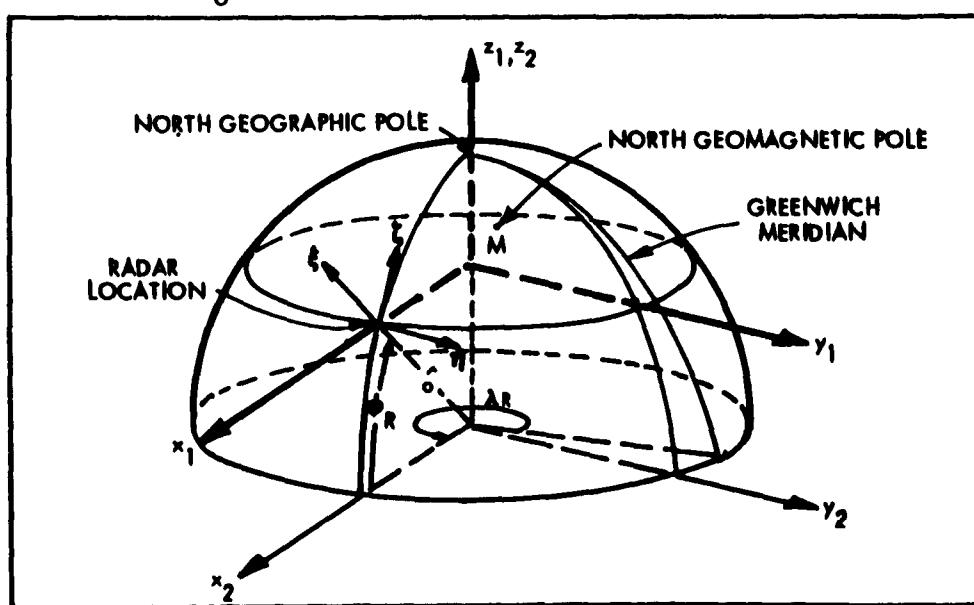


Figure 1b. Geometry of Coordinate Transformation

3. Let x_2, y_2, z_2 equal the orthogonal coordinate system with origin at the earth's center. Let the x_2 -axis be parallel to the x_1 -axis and z_2 coincide with w, hence also with z_1 . Then going from x_1, y_1, z_1 to x_2, y_2, z_2 by translation

$$\begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix} = \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ r_o \sin \varphi_R \end{pmatrix} = \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} + (c_i) \quad (48)$$

4. Let x_3, y_3, z_3 represent an orthogonal coordinate system with origin at the earth's center such that the x_3 -axis intersects the zero degree longitudinal geomagnetic meridian while the z_3 -axis coincides with w. Hence going from x_2, y_2, z_2 to x_3, y_3, z_3 by rotation about the z_2 -axis yields,

$$\begin{pmatrix} x_3 \\ y_3 \\ z_3 \end{pmatrix} = \begin{pmatrix} \cos(\lambda_M - \lambda_R) & \sin(\lambda_M - \lambda_R) & 0 \\ -\sin(\lambda_M - \lambda_R) & \cos(\lambda_M - \lambda_R) & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix} \quad (49)$$

$$\begin{pmatrix} x_3 \\ y_3 \\ z_3 \end{pmatrix} = (d_{ij}) \begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix}$$

where as before λ_M and φ_M represent the geographic longitude and latitude of the geomagnetic north pole M.

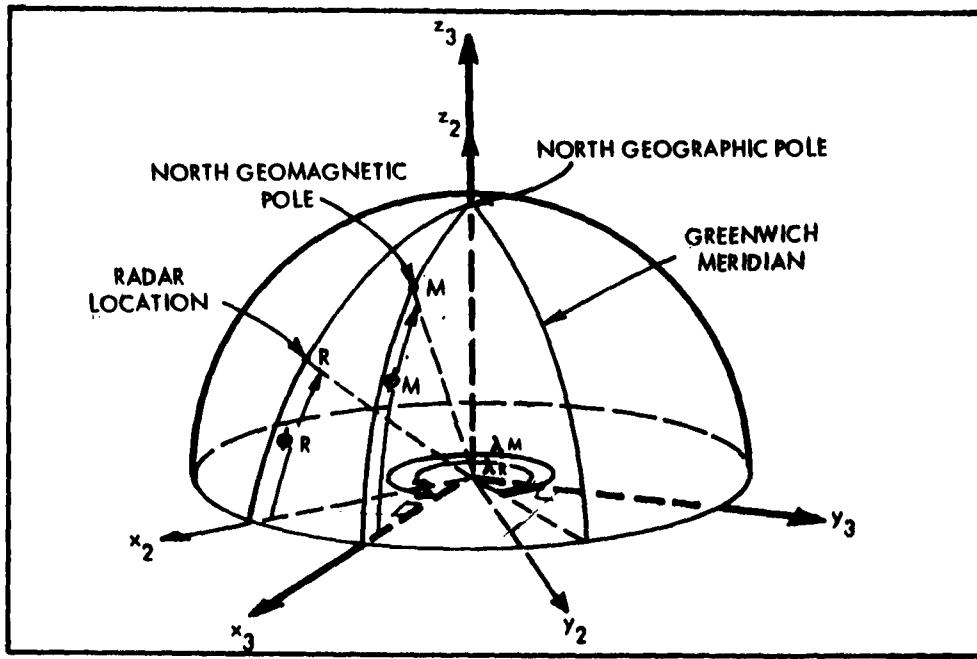


Figure 1c. Geometry of Coordinate Transformation

5. Let x , y , z represent an orthogonal coordinate system with origin at the earth's center. Let the x -axis pass through the great circle connecting the geographic and geomagnetic poles while the z -axis passes through the geomagnetic pole M . In going from x_3 , y_3 , z_3 to x , y , z by a rotation one obtains

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \sin\varphi_M & 0 & -\cos\varphi_M \\ 0 & 1 & 0 \\ \cos\varphi_M & 0 & \sin\varphi_M \end{pmatrix} \begin{pmatrix} x_3 \\ y_3 \\ z_3 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = (e_{ij}) \begin{pmatrix} x_3 \\ y_3 \\ z_3 \end{pmatrix} \quad (50)$$

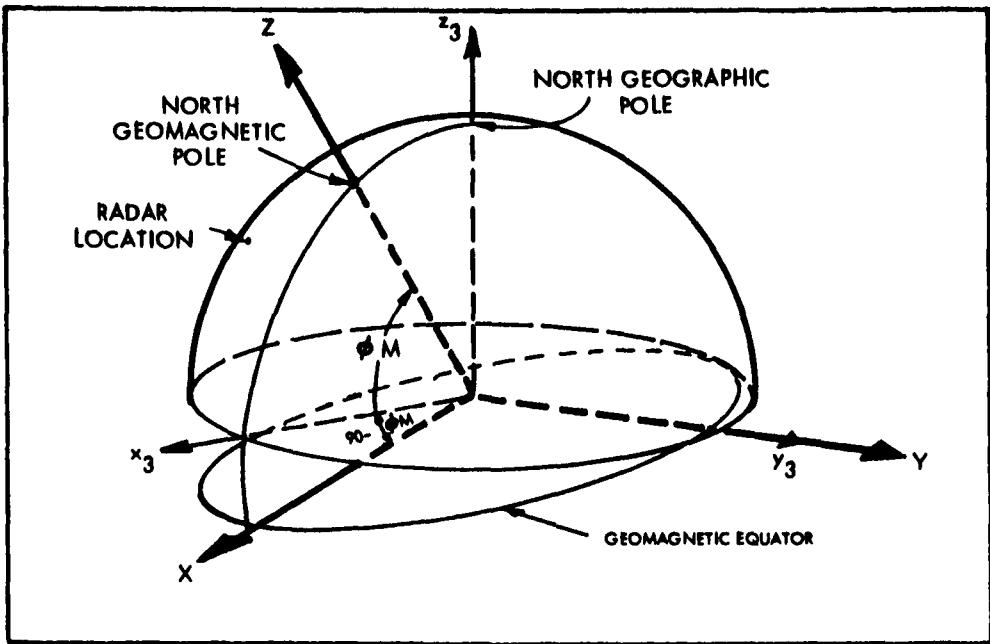


Figure 1d. Geometry of Coordinate Transformation

Hence by matrix multiplication one can transform from radar coordinates R, E, A to the earth centered coordinates x, y, z where the magnetic dipole axis of the earth coincides with the z-axis. From this coordinate system one can simply transform to the desired spherical coordinate system r, θ, φ.

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = (e_{ij}) \begin{pmatrix} x_3 \\ y_3 \\ z_3 \end{pmatrix} = (e_{ij})(d_{ij}) \begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix} = (e_{ij})(d_{ij}) \left\{ \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} + (c_i) \right\} \quad (51)$$

$$= (e_{ij})(d_{ij}) \left\{ (b_{ij}) \begin{pmatrix} \epsilon \\ \eta \\ \zeta \end{pmatrix} + (b_i + c_i) \right\}$$

$$(g_{ij}) = (e_{ij})(d_{ij})(b_{ij}) = (f_{ij})(b_{ij}) \quad (52)$$

$$(f_{ij}) = \begin{pmatrix} \sin\varphi_M \cos(\lambda_M - \lambda_R) & \sin\varphi_M \sin(\lambda_M - \lambda_R) & -\cos\varphi_M \\ -\sin(\lambda_M - \lambda_R) & \cos(\lambda_M - \lambda_R) & 0 \\ \cos\varphi_M \cos(\lambda_M - \lambda_R) & \cos\varphi_M \sin(\lambda_M - \lambda_R) & \sin\varphi_M \end{pmatrix} \quad (53)$$

$$g_{ij} = \begin{pmatrix} g_{11} & g_{12} & g_{13} \\ g_{21} & g_{22} & g_{23} \\ g_{31} & g_{32} & g_{33} \end{pmatrix} \quad (54)$$

$$g_{11} = (\cos\varphi_R \sin\varphi_M \cos(\lambda_M - \lambda_R) - \cos\varphi_M \sin\varphi_R)$$

$$g_{12} = \sin\varphi_M \sin(\lambda_M - \lambda_R)$$

$$g_{13} = (-\sin\varphi_R \sin\varphi_M \cos(\lambda_M - \lambda_R) - \cos\varphi_R \cos\varphi_M)$$

$$g_{21} = -\sin(\lambda_M - \lambda_R) \cos\varphi_R$$

$$g_{22} = \cos(\lambda_M - \lambda_R)$$

$$g_{23} = \sin\varphi_R \sin(\lambda_M - \lambda_R)$$

$$g_{31} = (\cos\varphi_R \cos\varphi_M \cos(\lambda_M - \lambda_R) + \sin\varphi_M \sin\varphi_R)$$

$$g_{32} = \cos\varphi_M \sin(\lambda_M - \lambda_R)$$

$$g_{33} = (-\sin\varphi_R \cos\varphi_M \cos(\lambda_M - \lambda_R) + \sin\varphi_M \cos\varphi_R)$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = R(g_{ij}) \begin{pmatrix} \sin E \\ \cos E \sin A \\ \cos E \cos A \end{pmatrix} + r_o(g_{il}) = r \begin{pmatrix} \sin \theta \cos \varphi \\ \sin \theta \sin \varphi \\ \cos \theta \end{pmatrix} \quad (55)$$

By use of these matrix transformations the Cartesian coordinates, (x , y , z), and from them the spherical coordinates, (r , θ , φ), of the earth centered geomagnetic coordinate system can be determined for the spatial starting point S and the earth bound transmitter R. These can be expressed as:

$$\begin{aligned} x_S &= [\cos(\lambda_M - \lambda_R) \sin\varphi_M \cos\varphi_R - \cos\varphi_M \sin\varphi_R] (R \sin E + r_o) \\ &+ [\sin(\lambda_M - \lambda_R) \sin\varphi_M] R \cos E \sin A \quad (56) \\ &- [\cos(\lambda_M - \lambda_R) \sin\varphi_M \sin\varphi_R + \cos\varphi_M \cos\varphi_R] R \cos E \cos A \end{aligned}$$

$$\begin{aligned} y_S &= [-\sin(\lambda_M - \lambda_R) \cos\varphi_R] (R \sin E + r_o) + [\cos(\lambda_M - \lambda_R)] R \cos E \sin A \\ &+ [\sin(\lambda_M - \lambda_R) \sin\varphi_R] R \cos E \cos A \quad (57) \end{aligned}$$

$$\begin{aligned} z_S &= [\cos(\lambda_M - \lambda_R) \cos\varphi_M \cos\varphi_R + \sin\varphi_M \sin\varphi_R] (R \sin E + r_o) \\ &+ [\sin(\lambda_M - \lambda_R) \cos\varphi_M] R \cos E \sin A \quad (58) \\ &- [\cos(\lambda_M - \lambda_R) \cos\varphi_M \sin\varphi_R - \sin\varphi_M \cos\varphi_R] R \cos E \cos A \end{aligned}$$

When $R = 0$, that is, for a point on the surface of the earth,

$$x_R = r_o [\cos(\lambda_M - \lambda_R) \sin\varphi_M \cos\varphi_R - \cos\varphi_M \sin\varphi_R] \quad (59)$$

$$y_R = -r_o \sin(\lambda_M - \lambda_R) \cos\varphi_R \quad (60)$$

$$z_R = r_o [\cos(\lambda_M - \lambda_R) \cos\varphi_M \cos\varphi_R + \sin\varphi_M \sin\varphi_R] \quad (61)$$

From simple trigonometric considerations (Figure 2a) it can be shown that the radar slant range, R , measured from the transmitter to the spatial starting point S is given by

$$R = -r_o \sin E + \sqrt{(r_o + h_S)^2 - r_o^2 \cos^2 E} \quad (62)$$

where h_S is the vertical height of the starting point above its projection, (point P) on the surface of the earth.

Equations 1 through 6 point out that in addition to these transformations, the components of the directed normal to the phase fronts, \vec{d} , at the starting point S are to be determined in this coordinate system. From spherical and plane trigonometric considerations, (Figure 2), it can be shown that these components are given by

$$\sigma_r = \sigma \cos e \quad (63)$$

$$\sigma_\theta = \sigma \sin e \cos a \quad (64)$$

$$\sigma_\varphi = -\sigma \sin e \sin a \quad (65)$$

Angle e can be evaluated directly by employing the law of sines. This yields

$$e = \sin^{-1} \left(\frac{r_o \cos E}{r_o + h_S} \right) \quad (66)$$

Angle a is the geomagnetic bearing angle (Figure 2) measured positive in a clockwise direction from geomagnetic north. By use of spherical trigonometry it is expressible by

$$a = \tan^{-1} \left[\frac{\sin(\Phi_S - \Phi_R) \sin \theta_R}{\cos \theta_S \sin \theta_R \cos(\Phi_S - \Phi_R) - \sin \theta_S \cos \theta_R} \right] \quad (67)$$

where Φ_R , θ_R , and Φ_S , θ_S are the geomagnetic longitudes and co-latitudes, respectively, of the radar transmitter, R, and the spatial starting point, S. The geomagnetic angles are obtained from the following expressions

$$\begin{aligned} \Phi_R &= \tan^{-1} \left(\frac{y_R}{x_R} \right) = \tan^{-1} \left(\frac{g_{21}}{g_{11}} \right) \\ &= \tan^{-1} \left[\frac{-\sin(\lambda_M - \lambda_R) \cos \varphi_R}{\cos(\lambda_M - \lambda_R) \sin \varphi_M \cos \varphi_R - \cos \varphi_M \sin \varphi_R} \right] \end{aligned} \quad (68)$$

$$\begin{aligned} \theta_R &= \cos^{-1} \left(\frac{z_R}{r_R} \right) = \cos^{-1} (g_{31}) \\ &= \cos^{-1} \left[\cos(\lambda_M - \lambda_R) \cos \varphi_M \cos \varphi_R + \sin \varphi_M \sin \varphi_R \right] \end{aligned} \quad (69)$$

$$\Phi_S = \tan^{-1}\left(\frac{y_S}{x_S}\right) \quad (70)$$

$$\theta_S = \cos^{-1} \frac{z_S}{r_o + h_S} \quad (71)$$

One additional useful expression can be obtained from these algebraic relations. The parameter is the angle ψ at the spatial starting point S which is defined by the inner product of the magnetic field vector and the wave normal. By the application of the sine and cosine laws to the geometry of Figure 2-a, it can be shown that

$$\psi = \cos^{-1} [-(\cos e \sin I + \sin e \cos I \cos a)] \quad (72)$$

where angle I is the magnetic inclination angle. The inclination angle⁹ is only a function of the geomagnetic latitude at the particular point in question.

$$I = \tan^{-1} [2 \cotan \theta_S] \quad (73)$$

Expressions arising from the inverse coordinate transformation, that is, transformation from the geomagnetic coordinates (r, θ, ϕ) to the radar coordinates R, E, A can be easily developed from these formulae. Although used in the computer program they will not be presented here.

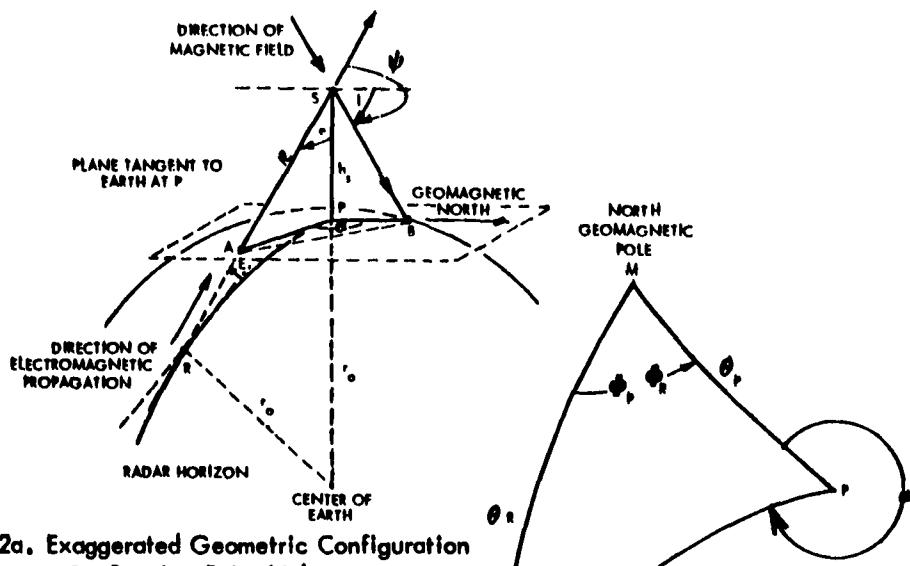


Figure 2a. Exaggerated Geometric Configuration
for Starting Point Values

Figure 2b. Spherical Triangle Illustrating the
Geomagnetic Bearing Angle

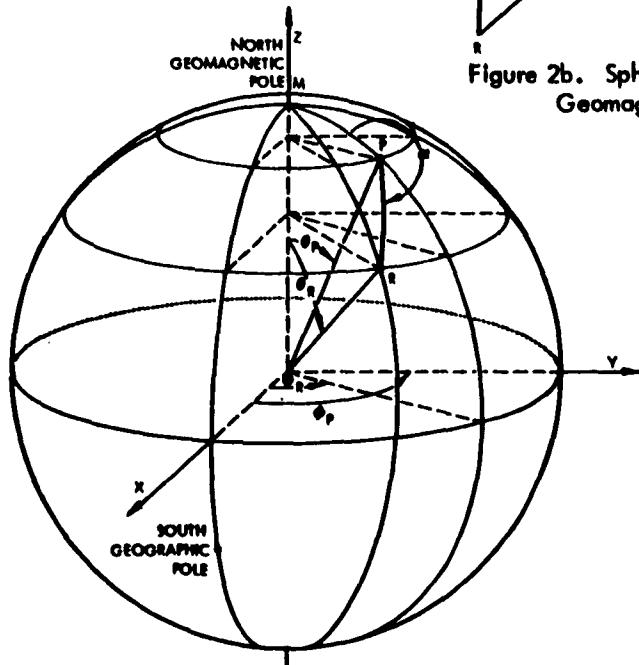


Figure 2c. Geomagnetic Coordinate System

Figure 2. Geometry for Starting Point and Geomagnetic Coordinate System

Nomenclature Used in Coordinate Transformation

r, θ, ϕ	spatial point in an earth centered spherical coordinate system
E	radar elevation angle
A	radar azimuth angle
$\phi_{R, M}$	geographic latitude of point R or M, respectively
$\lambda_{R, M}$	geographic longitude of point R or M, respectively
S	spatial starting point on nondeviated portion of ray
R	distance from electromagnetic wave transmitter to starting point
ϵ, η, ζ	set of orthogonal coordinates with origin on the surface of the earth at radar site
x_1, y_1, z_1	orthogonal coordinate system with origin on the earth's axis of rotation
x_2, y_2, z_2	orthogonal coordinate system with origin at earth's center
x_3, y_3, z_3	orthogonal coordinate system with origin at earth's center
w	z component of the geographic coordinate system (u, v, w)
h_S	height of starting point above its projection on the surface of the earth
r_o	radius of earth
a	geomagnetic bearing angle
ϕ_R, ϕ_S	geomagnetic longitudes of points R and S

θ_R, θ_S	geomagnetic co-latitudes of points R and S
$\sigma_r, \sigma_\theta, \sigma_\phi$	physical components of a vector of length μ , that is directed normal to the phase front
ψ	angle between magnetic field vector and the wave normal
I	angle of magnetic inclination

D. MODEL IONOSPHERE

As shown under Computational Procedure, the refractive index M and its spatial derivatives are dependent on the normalized density, X , and its spatial gradients, $\partial X / \partial r$, $\partial X / \partial \theta$, and $\partial X / \partial \varphi$. For an evaluation of these quantities an analytic model ionosphere can be chosen. One such ionospheric model that was found useful, is a spherical electron distribution as measured from a point in space. A reason for its selection is presented under Computational Results. Other uses as well as other ionospheric models are covered elsewhere^{7, 11}.

Let r_b , θ_b , φ_b represent the spatial location, B , of the center of the selected spherical ionosphere in the geomagnetic spherical coordinate system (r, θ, φ) . The values of these coordinates are obtainable from the specified geographic latitude, longitude and height above the earth surface of point B , in an analogous procedure as described for the transformation of coordinates of the spatial starting point S . Then the electron density at a spatial point r, θ, φ for an assumed spherical ionosphere can be written as:

$$N(r, \theta, \varphi, r_b, \theta_b, \varphi_b) = \frac{A}{\mathcal{R}^n} \quad (74)$$

where

$$\mathcal{R} = \left\{ (r_b \sin \theta_b \cos \varphi_b - r \sin \theta \cos \varphi)^2 + (r_b \sin \theta_b \sin \varphi_b - r \sin \theta \sin \varphi)^2 + (r_b \cos \theta_b - r \cos \theta)^2 \right\}^{1/2} = \left\{ X^2 + Y^2 + Z^2 \right\}^{1/2} \quad (75)$$

For this discussion A and n are appropriately chosen constants which give the desired electron density N (electrons/cc). By use of Equations 74 and 75, it is easily shown that the spatial electron density gradients can be expressed in the following manner,

$$\frac{\partial N}{\partial r} = \frac{nA}{\chi^{n+2}} \left[XP \sin \theta \cos \varphi + YP \sin \theta \sin \varphi + ZP \cos \theta \right] \quad (76)$$

$$\frac{\partial N}{\partial \theta} = \frac{nAr}{\chi^{n+2}} \left[XP \cos \theta \cos \varphi + YP \cos \theta \sin \varphi - ZP \sin \theta \right] \quad (77)$$

$$\frac{\partial N}{\partial \varphi} = \frac{nAr}{\chi^{n+2}} \left[-XP \sin \theta \sin \varphi + YP \sin \theta \cos \varphi \right] \quad (78)$$

Some results obtained by use of such a spherical ionospheric model will be discussed later.

E. MODEL OF EARTH'S MAGNETIC FIELD

Because magneto-ionic effects on the propagation of electromagnetic waves through an ionized medium are taken into account in the derivation of the equations under Computational Procedure, it is necessary to specify the normalized external magnetic field of the earth, \vec{Y} , its components Y_r , Y_θ , Y_ϕ and its spatial derivatives $\partial Y/\partial r$, $\partial Y/\partial \theta$, $\partial Y/\partial \phi$. It is known that the earth's magnetic field can be approximated by an earth centered magnetic dipole with its axis displaced such that the geographic longitude $\lambda_M = 70.1^\circ W$ and the geographic latitude $\Phi_M = 78.6^\circ N$. The magnetic potential, V , at a distant point from such a dipole is related to the magnetic moment, M , by the expression

$$V(r, \theta) = -\frac{M \cos \theta}{r^2} = -\frac{(Y_e r_o^3) \cos \theta}{r^2} \quad (79)$$

where r, θ are the geomagnetic coordinates of the spatial point irrespective of the coordinate ϕ and as before, r_o = radius of the earth. In this equation Y_e is the magnitude of the normalized magnetic field at the earth's surface on the magnetic equator. By use of this algebraic equation all the desired quantities can be derived. They are

$$Y = Y_e \left(\frac{r_o}{r}\right)^3 (1 + 3 \cos^2 \theta)^{1/2} \quad (80)$$

where, as previously defined, Y is the normalized magnitude of the earth's magnetic field vector $\vec{Y} = (eH/mc\omega) = \omega_c/\omega$

$$Y_r = 2 Y_e \left(\frac{r_o}{r}\right)^3 \cos \theta = \frac{Y}{\sqrt{1 + \frac{1}{4} \tan^2 \theta}} \quad (81)$$

$$Y_\theta = Y_e \left(\frac{r_o}{r}\right)^3 \sin \theta = \frac{1}{2} Y_r \tan \theta \quad (82)$$

$$Y_\varphi = \frac{\partial Y}{\partial \varphi} = 0 \quad (83)$$

$$\frac{\partial Y}{\partial r} = - \frac{3Y}{r} \quad (84)$$

$$\frac{\partial Y}{\partial \theta} = - \frac{3Y \sin \theta \cos \theta}{[1 + 3 \cos^2 \theta]} \quad (85)$$

F. MODEL OF ATMOSPHERIC COLLISION FREQUENCY

For some of the trial calculations the atmospheric collision frequency was found from assumed exponential variations of collision frequency with height. The atmosphere was radially stratified and an approximate exponential equation was curve-fitted to measured experimental data for each stratified region. Hence, for each region the following relations were used to obtain Z and $\partial Z / \partial r$, $\partial Z / \partial \theta$, $\partial Z / \partial \varphi$ that are required by the ray trace equations.

$$Z = \frac{v}{\omega} = ae^{-b(r - r_o)} \quad (86)$$

$$\frac{\partial Z}{\partial r} = -bZ \quad (87)$$

$$\frac{\partial Z}{\partial \theta} = \frac{\partial Z}{\partial \varphi} = 0 \quad (88)$$

The dependence of collision frequency on a localized temperature distribution and degree of ionization⁷ complicates these simple relations. These complications (as derived by D. Archer) as well as their effects will not be discussed at this time.

G. COMPUTATIONAL RESULTS

The preceding equations are only a summary of the required set which will permit the detailed calculation of a ray path in three-dimensional space. Because of this, it becomes clear that the only realistic approach to the solution of this problem is the utilization of computer techniques, otherwise, the welter of data that must be handled through use of numerical methods, is beyond effective human handling capacity. However, the development of a computer program which can perform countless number of calculations, poses the very difficult task of determining the correctness of a computed result. To simplify this "debugging" task the classical idea of elastic collision between charged particles was borrowed from nuclear physics. It has been shown¹⁰ that if the electron density falls off as the inverse square of the distance from the center of a spherical electron distribution ($N = A/\mathcal{R}^2$), various exact expressions can be obtained, since the ray equations at zero azimuth are integrable.

The geometry of such a distribution, as well as, three ray paths computed by use of the computer program are shown in Figure 3. In the figure the center of the sphere is located by the fixed coordinates (R_o, β_o) with respect to the radar. The derivations are made in two dimensions, hence only the two-dimensional coordinate system (ξ, ζ) is used. The ray path has an initial elevation angle E_1 and its distance of closest approach to the center of the refracting sphere is denoted by \mathcal{R}_o . The coordinates of any point on the ray path with respect to the center of the sphere are (\mathcal{R}, β) and with respect to the radar (R, E). Angle δ represents the amount of ray bending experienced by the ray passing through the refractive medium. Under these conditions Archer¹⁰ has shown that the angle δ is given by

$$\delta = \pi - 2 \left[\gamma_o + \mu (\mathcal{R}_o) \cos^{-1} \left(\frac{\mathcal{R}_o}{R_o} \right) \right] \quad (89)$$

The three plotted rays have actually a small third dimensional component. Because of this, the accuracy of these plotted rays is approximately (+3, -2) percent.

As shown by the tabulated results in Table 1, the computed deviation angle δ_c , arising from ray trace results, agrees very well with the calculated angle, δ , obtained by use of Equation 89. These computer

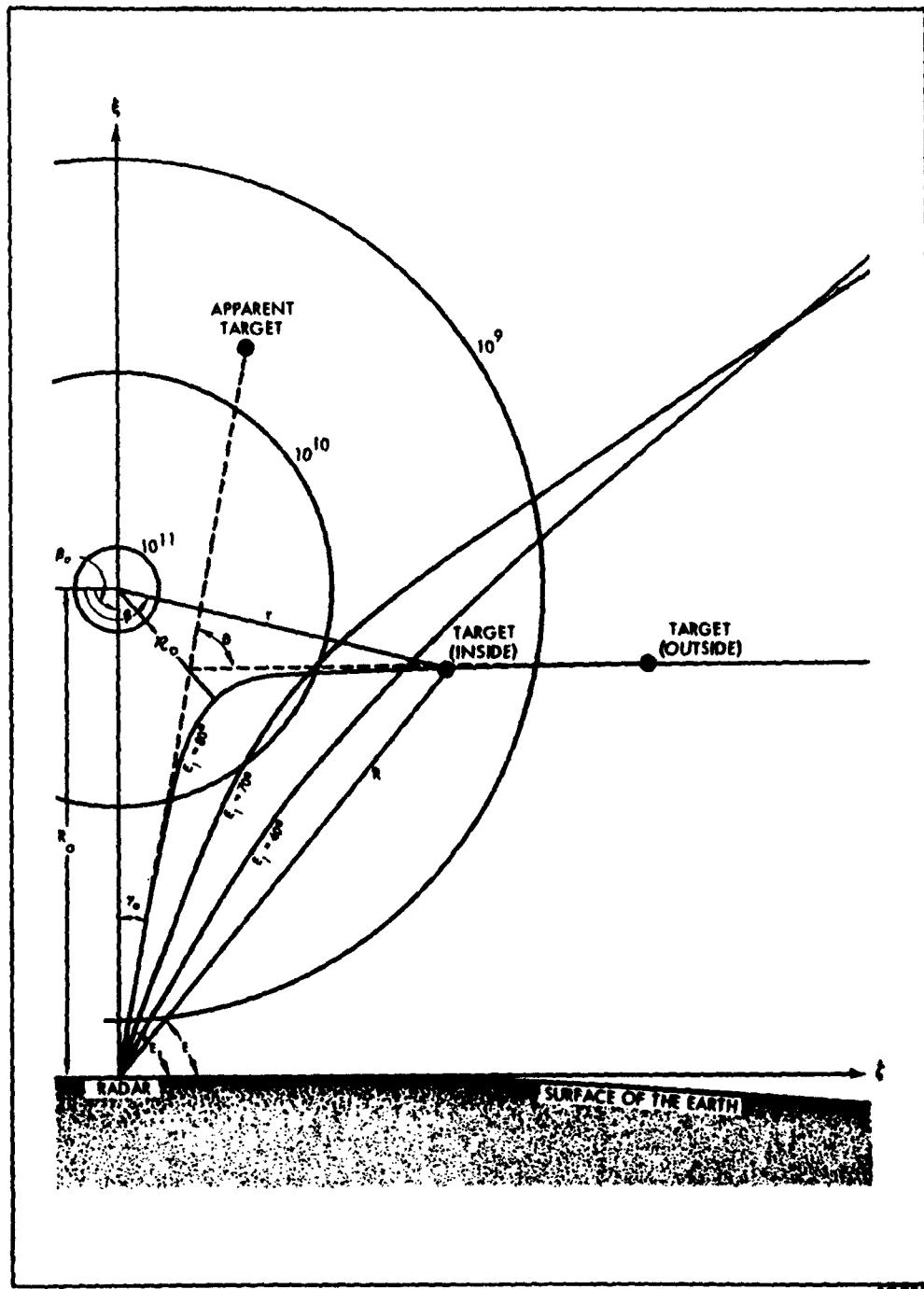


Figure 3. Geometry for Refraction by Spherically Stratified Region

calculations were performed under a large error upper bound condition ($\bar{E} = 10^{-3}$, see description of Subroutine INT). The agreement is improved by a variation of this error condition. Additional results will not be presented here. Presented elsewhere¹² is the influence of \bar{E} on computed results using these numerical methods as applied to the study of ionization-deionization phenomena.

Radar Elevation Angle - E Degrees	Bending Angle - δ from Equation 89 Degrees	Bending Angle - δ_c Ray Trace Program Degrees
60	19.5	20
70	35.2	37
80	77.5	78

Table 1
Comparison of Total Ray Bending Angle

As an additional illustration, refractive errors through a particular spherical ionized region have been computed in detail to illustrate the concepts discussed and to indicate the kinds of refractive errors that could arise. The electron density contours of this ionization model are defined by Equation 74 where $A = 10^{33}$ and $n = 12$. The distance from the center of the spherical ionosphere is measured in kilometers. The center of the model is located at an elevation angle, E, of 30 degrees, zero degree azimuth angle, and 564 kilometers slant range as measured from the radar site.

Figure 4 shows the relation between the radar and ionization model in the plane of zero azimuth. Also shown are the ray paths for rays leaving the radar at several elevation angles. A frequency of one kilomegacycle was used in determining the refraction of the electromagnetic wave propagation vector. Because the electron density increases rapidly near the center of the model, there is significant bending of the ray path.

The refraction becomes so severe as the elevation angle of radar rays approach the elevation angle of the ionization center with respect to the radar, that there is a region (shown by half tones) into which no radar ray penetrates, hence, radar "blackout" is achieved. In three dimensions this blackout region is a cone in which a target is shielded from the radar. Because rays near this region intersect each other, two elevation angle paths to the same target exist, so multiple targets may be visible.

If the ray path is not in the zero azimuth plane, the amount of elevation error, or azimuth error, is a function of the location of the target. The elevation and azimuth errors for a target located at a slant range of 1200 kilometers have been computed as a function of radar elevation and azimuth angles. These are summarized in Figure 5, in which contours of constant elevation error, ΔE , in one quadrant and constant azimuth error, ΔA , in another quadrant as a function of the ray direction at the radar site are given. The contours have been terminated at a total bearing error of about 10 degrees. Due to symmetry the errors in the other quadrants are just the mirror image of the quadrants shown.

Nomenclature Used for Computational Results

E_1	initial elevation angle made by ray path
R_o	distance of closest approach between ray path and center of sphere
(ρ, β)	coordinates of any point on the ray path with respect to the center of the sphere
(R, E)	coordinates of any point on the ray path with respect to the radar
R_o	distance between the radar and center of the sphere
γ_o	apparent bearing angle

SECTION III

COMPUTER PROGRAM FOR THREE-DIMENSIONAL RAY-TRACING

Figure 6 schematically describes the computer program that was developed for three-dimensional ray tracing. As illustrated, the computer program is a composite of a group of subprograms. Because each subroutine is an entity in itself, the improvement of the entire program can be performed by the variation of each subprogram.

For the creation of this program the FORTRAN language¹³ was used wherever possible. FORTRAN is an automatic coding system for the IBM-704/709/7090 Data Processing Computer System that was designed for scientific application. Although there are limitations to FORTRAN, nevertheless, 1) it is at present the only language for scientific use, that is accepted by most existing large computer systems, and 2) it is simple and therefore without much effort, permits the elimination of the programmer, thus leaving the design of logical computer decisions, to the formulator of the scientific problem. The program has been written to operate "in or out" of the FORTRAN MONITOR CONTROL SYSTEM.

Except for the RINDEX subroutine the program has been divided into small, simple Functions and Subroutines to facilitate understanding. In the development of the program, concentration was mainly on obtaining a correct working program, as soon as possible, and not on optimization or clarity of output results. These tasks are left for future development.

The computer program consists of the following parts:

- | | |
|-----------------|-----------|
| 1) Main Program | RAY TRACE |
| 2) Function | SLANTR |
| 3) Function | QATAN |
| 4) Function | ARCOS |

5) Subroutine	COORD
6) Subroutine	DAUX
7) Subroutines	INT and INTM
8) Subroutine	RINDEX
9) Subroutine	ELECTX
10) Subroutine	BIGR
11) Subroutine	MAGY
12) Subroutine	COLFRZ
13) Subroutine	RCOORD
14) Subroutine	OUTONE
15) Subroutine	OUTPUT

These functions and subroutines are used to obtain numerical values for those variables which cannot be defined by only one arithmetic statement. In addition to these subprograms certain statements in the FORTRAN language cause the inclusion in the object program of the necessary input and output routines, as well as, various library functions and subroutines in relocatable binary form that are available on the FORTRAN MASTER LIBRARY TAPE. The names and locations of these necessary routines are given in the "storage map" of the arrangement of storage location in the object program that is compiled from a FORTRAN source program. These "maps" follow the listings of each source program. These added routines will not be discussed in this report. Following a brief description of the function of the main program and its associated subprograms, the necessary input data for a sample calculation is given with a description of the output. Some of the results listed in this output led to the graphical results presented in Figures 3, 4 and 5.

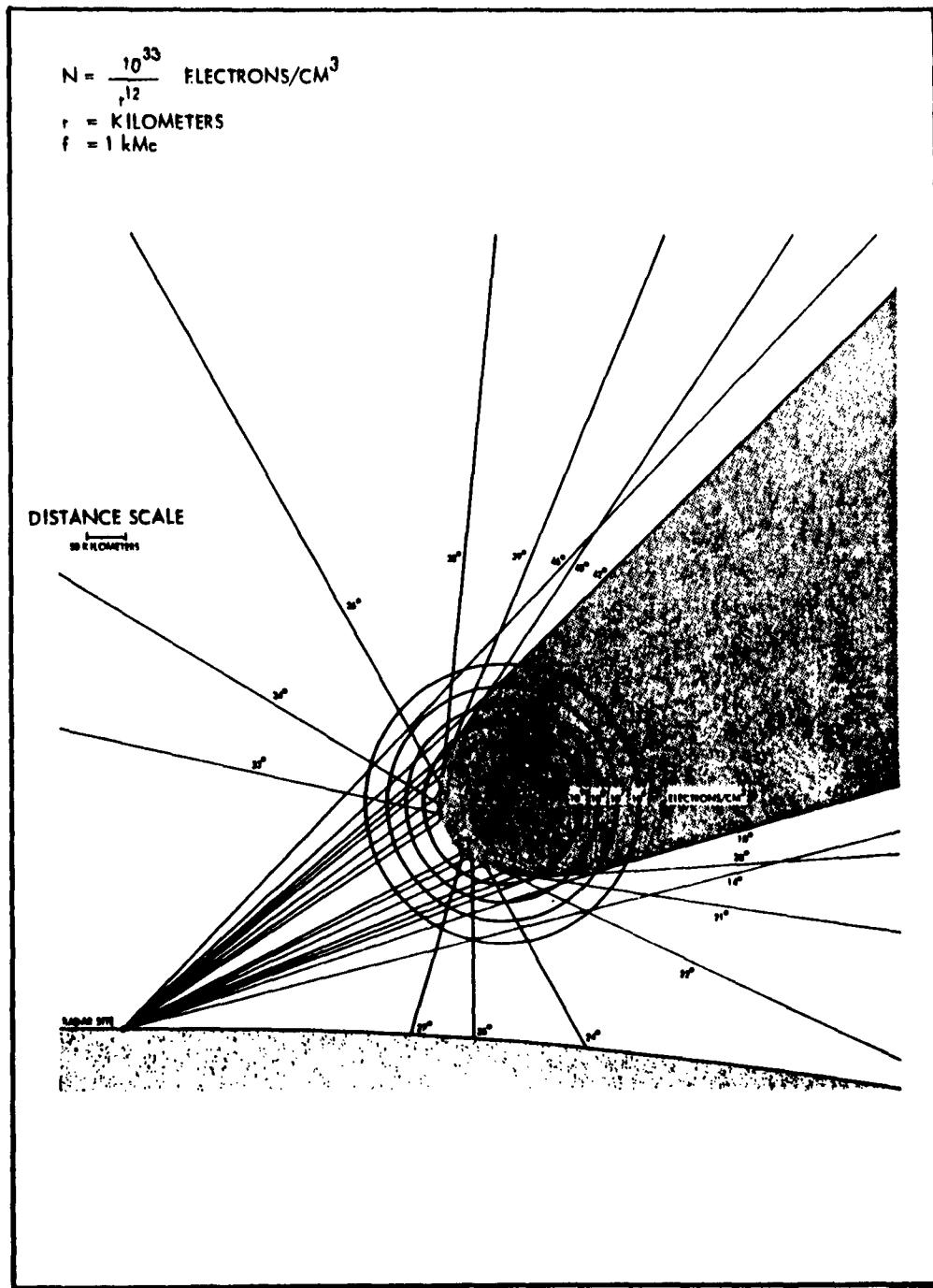


Figure 4. Radar Propagation Paths through Spherically Ionized Region

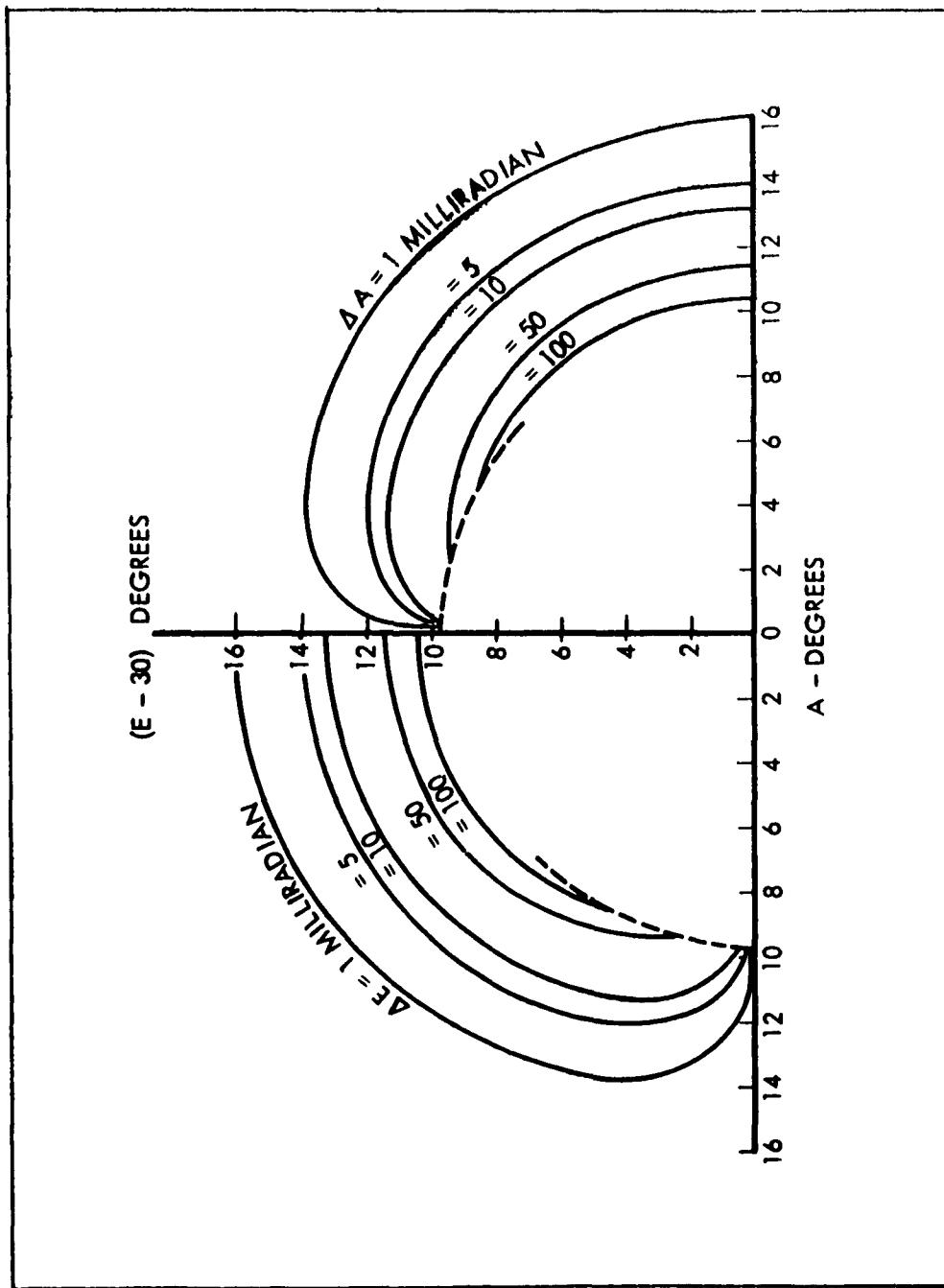


Figure 5. Elevation and Azimuth Errors for Propagation through
a Spherical Model - $f = 1 \text{ kMc}$

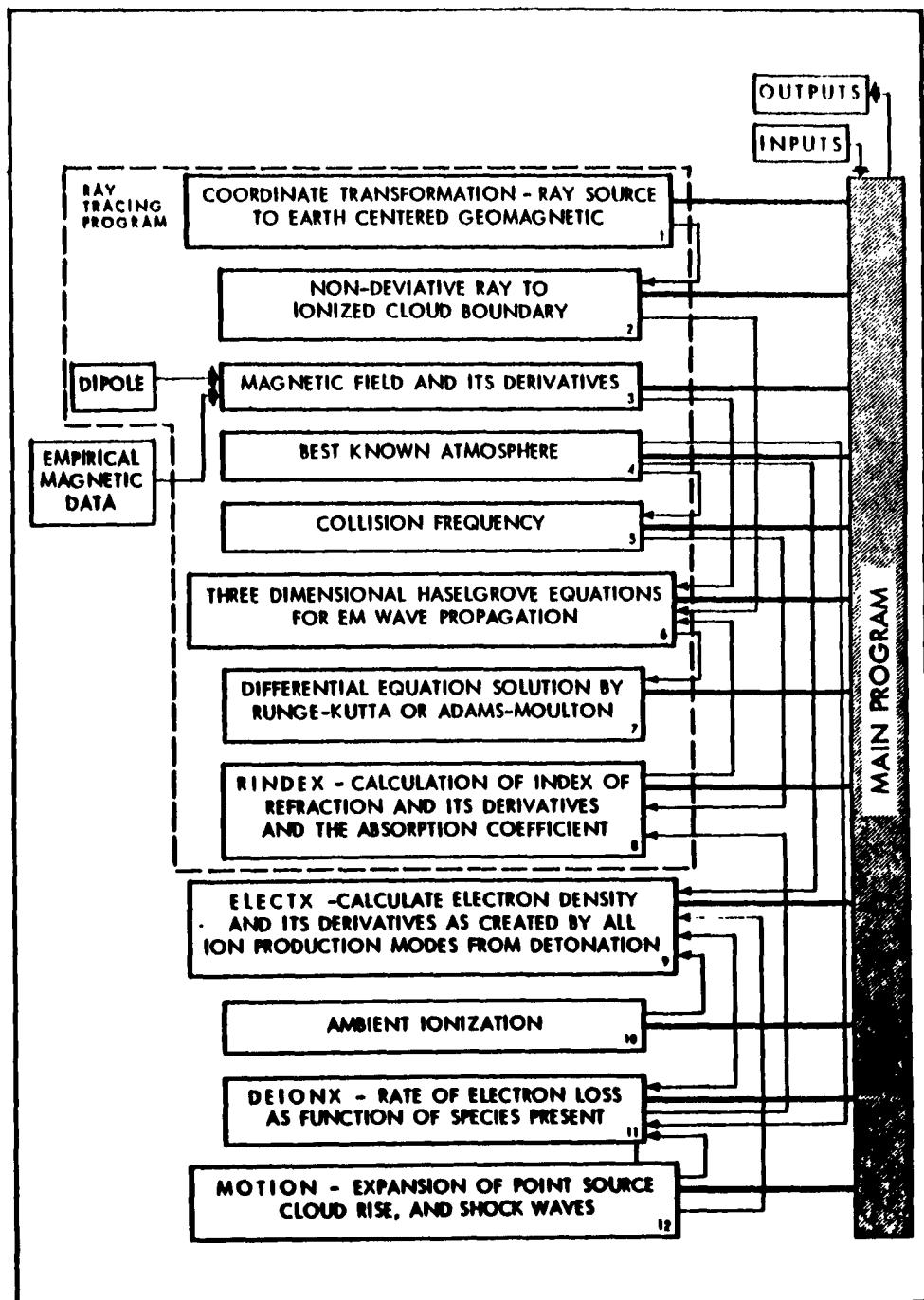


Figure 6. Block Diagram of a Possible Computer Program

A. MAIN PROGRAM RAY TRACE

The main program's function is to act as a master control of the logical flow necessary for the execution of the numerical methods. In addition, it is responsible for obtaining the necessary data, initializing the required starting conditions, performing the desired controlled printouts of computed results, and determining the condition for termination of the given computations. For initializing the starting conditions the main program requires the reading into storage of the following information in the format illustrated under INPUT, Table 4.

CARD 1

RECORD: This can be any desired information consisting of 72 alphanumeric characters that will serve to identify the calculations.

CARD 2

Contains the values of ID, KWIT. ID is an integer that can be used for identifying the calculation if the same CARD 1 is used. KWIT: On completion of the calculations in progress, the computer will check if additional problems are to be performed. Thus if KWIT=59 the computer will want to read a new W vector (CARD 3 --- onward); if KWIT=66, the computer will want to read a new CARD 1, CARD 2, and new W vector (CARD 3 --- onward); if KWIT equals any other integer the computer will PAUSE 44444.

CARD 3 onward

This card and all following cards describe the value of each component of the W vector that is not zero. As shown under INPUT, the first three columns of the card are for the integer that describes the W vector component. The next fourteen columns of the card are for the value of the W vector component. The number of these cards is variable since on completion of one calculation, often only one component of the W vector

CARD 3 (continued)

is to be changed for the next computation. The W vector can be read in any order.

LAST DATA CARD

This card follows the last card describing the W vector. It is any negative integer listed in the first three columns of a card. It transfers the computer out of the read mode to the location beginning the ray trace calculations.

SENSE SWITCH 1

The program is designed to calculate first the ordinary ray path and then the extraordinary ray path. SENSE SWITCH 1 DOWN will eliminate the calculation of the extraordinary ray path.

SENSE SWITCH 2

In DOWN position will permit the calculation of the extraordinary ray path and eliminate the calculation of the ordinary ray path.

SENSE SWITCH 3

Placing this SENSE SWITCH 3 DOWN will terminate the calculation on completion of the ray path calculations in progress.

SENSE SWITCH 4

Placing SENSE SWITCH 4 DOWN will cause the computer to check if SENSE SWITCH 6 is DOWN. If it is down the computer will terminate calculations immediately.

SENSE SWITCH 6

It is desirable to follow the course of any calculation on a computer. SENSE SWITCH 6 DOWN will print on-line, the total number of numerical integrations completed up to this point, integration mesh size, length of independent variable τ , height above surface of the earth (km), θ, Ψ (in degrees), $\sigma_r, \sigma_\theta, \sigma_\phi$, μ , κ , distance from the ion source center (km), value of the normalized electron density X.

SENSE LIGHT 2

When SENSE SWITCHES 1 and 2 are DOWN, then both ordinary and extraordinary ray paths are being calculated. When SENSE LIGHT 2 is ON, then the calculation is determining the ordinary ray path. When it is OFF, then the extraordinary ray path is being calculated.

PAUSE 17171

If the computer halts with this octal number in the address field of the STORAGE REGISTER it signifies that SENSE SWITCHES 1 and 2 are in the UP position and the problem is undefined. SENSE SWITCHES 1 or 2, or 1 and 2 are to be placed DOWN depending if only the ordinary, the extraordinary, or both ray paths are to be calculated. Following the definition of the problem, pushing START key will cause calculations to resume.

PAUSE 66666

The computer halts with this octal number in the STORAGE REGISTER just prior to beginning calculations. If the MONITOR system is used it permits the operator to know when it has left the MONITOR system and the SENSE SWITCHES can be changed as needed by the problem.

PAUSE 44444

The computer halts with this octal number in the STORAGE REGISTER on completion of all the necessary calculations specified by the INPUT data. It permits the operator to reset the desired sense switches for the MONITOR system. Pressing START will cause the computer to exit from the program to the MONITOR system.

Table 2 contains the nomenclature that describes some of the components of the V vector and the components of the W vector.

V(2)	independent variable τ
V(3)	initial step size input $\Delta\tau$
V(4)	radius from center of earth r
V(5)	variable angle θ
V(6)	variable angle Φ
V(7)	σ_r
V(8)	σ_θ
V(9)	σ_Φ
V(10)	optical path length one way s
V(11)	time one way T
V(12)	A absorption
V(13)	$dr/d\tau$
V(14)	$d\theta/d\tau$
V(15)	$d\Phi/d\tau$
V(16)	$d\sigma_r/d\tau$
V(17)	$d\sigma_\theta/d\tau$
V(18)	$d\sigma_\Phi/d\tau$
V(19)	$ds/d\tau$
V(20)	$dT/d\tau$
V(21)	$dA/d\tau$

Table 2. Nomenclature Describing the V and W Vectors.
 (Page 1 of 6)

W(1)	refractive index μ
W(2)	imaginary part of complex phase refractive index κ
W(3)	radar transmitter angular frequency ω
W(4)	$\partial\mu/\partial\sigma_r$
W(5)	$\partial\mu/\partial\sigma_\theta$
W(6)	$\partial\mu/\partial\sigma_\phi$
W(7)	$\partial\mu/\partial r$
W(8)	$\partial\mu/\partial\theta$
W(9)	$\partial\mu/\partial\phi$
W(10)	$\partial\mu/\partial\psi$
W(11)	$\partial\mu/\partial\omega$
W(12)	unassigned for this program
W(13)	geographic longitudinal angle λ_M of geomagnetic north pole measured east of Greenwich Meridian (degrees)
W(14)	angle λ_R measured as W(13) in degrees
W(15)	angle Φ_M geographic latitude of geomagnetic north-pole measured plus from geographic equator north
W(16)	angle Φ_R geographic latitude of radar (degrees)
W(17)	radar elevation angle E (degrees)
W(18)	radar azimuth bearing angle angle A (degrees)
W(19)	r_o radius of the earth (km)
W(20)	h_S height of starting point above surface of earth

Table 2. Nomenclature Describing the V and W Vectors.
(Page 2 of 6)

W(21)	angle Φ_B of ionization source measured as W(15) (degrees)
W(22)	longitudinal angle λ_B of source measured as W(13)
W(23)	h_B height of ionization source center above earth surface
W(24)	$\Delta\tau$ initial mesh size of variable
W(25)	Y_e normalized equator magnetic field on earth's surface at the geomagnetic equator
W(26)	a constant determining collision frequency
W(27)	b constant in exponent determining collision frequency
W(28)	range (km) = distance from ionization source center to spatial point (r, θ, Φ) =
W(29)	cosine of angle makes with the vertical through center of the ionizing source
W(30)	R_b radial distance from earth's center to center of ionizing source
W(31)	x geomagnetic coordinate of source
W(32)	y geomagnetic coordinate of source
W(33)	z geomagnetic coordinate of source
W(34)	A = constant in A/R^n determining electron density
W(35)	n = exponent in A/R^n equation
W(36)	unassigned for this program
W(37)	unassigned for this program
W(38)	plasma angular frequency cycles/sec

Table 2. Nomenclature Describing the V and W Vectors.
(Page 3 of 6)

W(39)	N_e in ion pairs/cc
W(40)	maximum $r = V(4)$ to be considered in this calculation
W(41)	A1 = vector in INTM routine If W(41) = 0 routine will use predictor corrector with variable $V'(24)$ If W(41) = 2 will use Runge-Kutta with fixed W(24) If W(41) = 3 will use predictor-corrector with fixed W(24) If W(41) = 1 or 2 then W(42) through W(47) are ignored but must have some value. If W(41) = 0 they are not ignored
W(42)	A2 = E upper bound on truncation error. See upper bound Equation (10) Appendix A in the INT and INTM subroutine
W(43)	A3 = M is value from which lower bound E is calculated $LBE = UBE/M$ in subroutine INT
W(44)	A4 = A as used in truncation error test EQ (10) in subroutine INT
W(45)	A5 = upper bound on mesh size (If = 0 no upper bound as long as within error range)
W(46)	A6 = lower bound on mesh size (If = 0 lower bound = 0)
W(47)	A7 = β , that is, 0 is less than β less than 1. It is used to decrease or increase mesh size by dividing or multiplying current integration mesh being used
W(48)	smallest attenuation to be considered
W(49)	initial refraction index = W(1)
W(50)	initial absorption kappa κ = W(2)

Table 2. Nomenclature Describing the V and W Vectors.
(Page 4 of 6)

W(51)	initial attenuation = A
W(52)	x_R (km) (Radar coordinate in geomagnetic coordinate system)
W(53)	y_R (km) (Radar coordinate in geomagnetic coordinate system)
W(54)	z_R (km) (Radar coordinate in geomagnetic coordinate system)
W(55)	$R = \sqrt{(x_R - x)^2 + (y_R - y)^2 + (z_R - z)^2}$ (km)
W(56)	$\Delta R = c\tau - W(55) = (2.99791 \times 10^5)[V(11)] - W(55)$ km
W(57)	new elevation angle E in degrees
W(58)	$\Delta E = W(57) - W(17)$ degrees
W(59)	$2[(W(1))(W(2))] / W(1)^2 - W(2)^2$
W(60)	slant range at r, θ, φ
W(61)	angle A at r, θ, φ
W(62)	elevation angle E at r, θ, φ
W(63)	assigned value to k; if 1 then control is on radius; if 2 then control is on range W(28); if 3 then control is on slant range W(60)
W(64)	value of Z
W(65)	value of Y
W(66)	value of X
W(67)	location of sign which determines the calculation for ordinary or extraordinary ray
W(68)	value of V(4) above which RINDEX is to print R vector

Table 2. Nomenclature Describing the V and W Vectors.
(Page 5 of 6)

W(69)	value W(1) below which R vector is printed if W(68) = 0
W(70)	number of performed integrations
W(71)	a in COLFRZ 100-200 km
W(72)	b in COLFRZ 100-200 km
W(73)	a in COLFRZ 200-300 km
W(74)	b in COLFRZ 200-300 km
W(75)	a in COLFRZ 300-400 km
W(76)	b in COLFRZ 300-400 km
W(77) to W(250)	unassigned in this program

Table 2. Nomenclature Describing the V and W Vectors.
(Page 6 of 6)

```

MAIN PROGRAM - PAY TRACE JANUARY 20. 1961 IBM-7090
C SWITCH 1 DOWN CALCULATE ORDINARY RAY ONLY
C SWITCH 2 DOWN CALCULATE EXTRA-ORDINARY RAY
C SENSE SWITCH 3 DOWN WILL EXIT AFTER COMPLETING THIS RAY
C SENSE SWITCH 4 WITH SENSE SWITCH 6 DOWN WILL EXIT
C SENSE SWITCH 6 DOWN PERMITS ON-LINE PRINTING OF CALCULATIONS IN
C PROGRESS INT.VC3,V(2),V(4)-RO,V(5),V(6),V(7),V(8),V(9),W(1),W(2),
C W(3),W(6)
PAUSE 66666 READY TO EXECUTE SET SENSE SWITCHES
PAUSE 17171 FORGOT TO SET SS1 AND SS2 FOR PROPER RAY CALCULATION
PAUSE 44444 READY TO EXIT SET SENSE SWITCHES FOR MONITOR
COMMON RECORD,Y,M,N,X1,XN,ZN,G
DIMENSION RECORD(12),V(111),W(250)
DIMENSION XNC7),ZN(7),ZN(8),GC3,3)
PRINT 111
PAUSE 66666
PRINT 114
66 READ INPUT TAPE 5.E7.(RECORD(1), I=1,12)
FORMAT 612A6)
67 READ INPUT TAPE 5.113.1D.KWIT
READ INPUT TAPE 5.60.K,DATA
FORMAT 613,E14.7 )
68 IF(KO) 100,62,62
69 IF(CY-13)*(K-14)*(K-15)*(K-16)*(K-17)*(K-18)*(K-19)*(K-21)*(K-22)) 64,
163,64
70 DATA = DATA 57.29578
N(1)= DATA
6+-----+
65 GO TO 59
100 SENSE LIGHT 0
101 IF (SENSE SWITCH 1) 122,106
102 SENSE LIGHT 1
103 IF (SENSE SWITCH 2) 104,1
104 SENSE LIGHT 2
105 GO TO 1
106 IF (SENSE SWITCH 2) 104,107
107 PRINT 110
PAUSE 17171
108 GO TO 100
110 FORMAT 6SM0 SET SS1 AND SS2 FOR PROBLEM BEING CONSIDERED
111 FORMAT 54H0 SET ALL SENSE SWITCHES IN POSITION DURING EXECUTE)
112 FORMAT 68H0 SET SENSE SWITCHES FOR MONITOR - READY TO EXIT)
113 FORMAT (216)
114 FORMAT 6MH0 INT.6HDLV.5X,4HVC20.4X,7HV.42-RO,3X,2HTHETASX,3HPhi,
15.4HSIGR.3X,8HSIGTHETR.6X,6HS1GPhi.5X,4HW(1),6X,4HW(2),5X,SHRANGE
2.7X,1HX)
115 FORMAT (1H0,15,1P7E9.2,3H .1P6E9.2,1H ,1P6E9.2,2H ,1P3E9.2)
1 APG1R = SINFC W(132-W(142)
APG2R = COSFC W(132-W(142)
APG3R = SINFC W(152)
APG4R = COSFC W(152)
APG5R = SINFC W(162)
APG6R = COSFC W(162)
APG7R = SINFC W(172)
APG8R = COSFC W(172)
APG9R = SINFC W(182)
APG10R = COSFC W(182)
IM = W(63)
PI = 3.1415927

```

```

5.1.10 = ARG2P*ARG3M*ARG6R - ARG4M*ARG5R
G1,2 = ARG1R*ARG3M
G1,3 = ARG2R*ARG3M*ARG2P - ARG5R*ARG4M
G1,4 = ARG1R*ARG6R
G2,1 = ARG2R
G2,2 = ARG5R*ARG1P
G2,3 = ARG5R*ARG4M*ARG2R + ARG3M*ARG6R
G2,4 = ARG4M*ARG1R
G2,5 = ARG5R*ARG4M*ARG2P - ARG3M*ARG5R
XH1D = G2,2)*G(3,3)
XH2D = G(1,2)*G(2,3)
XH3D = G(1,3)*G(3,2)
XH4D = G(1,3)*G(2,2)
XH5D = G(1,2)*G(3,3)
XH6D = G(3,2)*G(2,3)
XH7D = W(19)*G(1,1)
YH1D = G(1,1)*G(3,3)
YH2D = G(1,3)*G(3,1)
YH3D = G(1,3)*G(2,1)
YH4D = G(1,3)*G(3,1)
YH5D = G(2,1)*G(3,3)
YH6D = G(1,1)*G(2,3)
YH7D = W(19)*G(2,1)
ZH1D = G(1,1)*G(2,2)
ZH2D = G(1,2)*G(3,1)
ZH3D = G(2,1)*G(3,2)
ZH4D = G(2,2)*G(3,1)
ZH5D = G(1,2)*G(2,1)
ZH6D = G(1,1)*G(3,2)
ZH7D = W(19)*G(3,1)
DENM = ZNC(3)*G(3,3) + ZNC(2)*G(2,3) + ZNC(3)*G(1,3) - G(1,3)*ZNC(4)
! - ZNC(5)*G(3,3) - G(2,3)*ZNC(6)
ZN(8) = DENM
SRSP = SLANTR CARG7R, ARG9R, W(19), WC2D)
2 N = 9
3 OCALL COORD CXSP, YSP, ZSP, ARG1R, ARG2R, ARG3M, ARG4M, ARG5R, ARG6R, ARG7R,
1 ARG8R, ARG9R, ARG10R, SRSP, W(19))
4 RSP = SQRT(CXSP**2+YSP**2+ZSP**2)
5 QSP = ZSP/RSP
6 THETSp = ARCCOS(QSP)
7 PH1SP = QATAN(CXSP, YSP)
8 ZERO = 0.0
9 OCALL COORD CXR, YR, ZR, ARG1R, ARG2R, ARG3M, ARG4M, ARG5R, ARG6R, ARG7R,
10 ARG8R, ARG9R, ARG10R, ZERO, W(19))
10 OR = ZR W(19)
WC52 = XR
WC53 = YR
WC54 = ZR
11 THETR = ARCCOS(QR)
12 PHIR = QATAN(XR, YR)
13 ARGSE = CCCW(19))/-RSP)+ARG8R)
14 TERM2 = COSP*QSP)
15 ARG18 = SINF(CYC(13)) - Y(22))
ARG2B=COSFCYC(13)-W(22))
ARG5S=SINF(CYC(21))
ARG6B=COSFCYC(21))
16 OCALL COORD CXD, YD, ZD, ARG1B, ARG2B, ARG3B, ARG4M, ARG5B, ARG6B, ZERO,
17 ZERO, ZERO, ZERO, ZEPO, W(19))

```

RM 61TMP-32

```

17      WC200=WC190+WC23)
18      CB = 20*WC190
19      TPHTS = ARCCOS(CB)
20      PHTR = ATAN(CX0,YG)
21      U(1)= C20*WC190*WC200
22      U(2)= (AD-M190)*WC200
23      U(3)= C20*WC190*WC200
24      TPHM= CPHISP-PHTR,
25      TPHS = SQRTFC(1.0-QR*QR)
26      UTERMS = TERMS* SIN(CTERM4)
27      TERM7 = QSP*TERMS*COSFC(TPHM4)- SQRTFC(1.0-TERM2)*QR
28      ANALPH = DATA(CTERM7,TERM6),
29      STEP = H(1)*SQRTFC(1.0-ARGSE**2)
30      PI = 3.1415927
31      GAMMA = ANALPH - PI
32      SIGTHE = CM(C1)* ARGSE* COSFC(GAMMA)
33      SIGPHI = -(CN(C1))*ARGSE*SINFC(GAMMA))
34      H(2) = W(41)
35      IF CSENSE LIGHT 1> 32. 34
36      GO TO 36
37      IF CSENSE LIGHT 2> 35,290
38      SIGN = 1.0
39      SIGN 2 -1.0
40      IF CSENSE SWITCH 3> 37,39
41      PRINT 112
42      PAUSE 49444
43      CALL EXIT
44      NC12= WC49)
45      NC57= SIGN
46      NC70= 0.0
47      UC22= SRSP
48      UC32= WC24)
49      UC42= RSP
50      UC52= THETSP
51      UC61= PHISP
52      UC71= SIGR
53      UC82= SIGTHE
54      UC90= SIGPHI
55      UC102= SRSP,
56      UC110= CSRSP, 3.0E 5>
57      UC120= WC51)
58      WC20 = WC50)
59      I0 = 20 + 1
60      CALL OUTONE(SIGN, ID,
61      CALL INT CU, N, NR ,WC42),WC33),WC44),WC45),WC46),WC47)
62      IFCN(4) = WC190/31.3154
63      CALL INTW
64      IF CSENSE SWITCH 6>16,117
65      I17 IF WC120>13.119.56
66      TE1 = UC50+57.29578
67      TE2 = WC50+57.29578
68      INT = WC70)
69      PRINT 118
70      FORMAT(C24H0, WC12) LE NEGATIVE OR ZERO)
71      TE3 = WC42-WC190)
72      IF CSENSE SWITCH 6>16,117
73      I17 IF WC120>13.119.56
74      PRINT 119
75      FORMAT(C24H0, WC12) LE NEGATIVE OR ZERO)
76      TE1 = UC50+57.29578
77      TE2 = WC50+57.29578
78      INT = WC70)
79      PRINT 115, INT, UC3), UC22), TE3, TE1, TE2, UC71, UC80), UC90, WC12, WC22), WC28
80      I, WC50)
81      CALL OUTPUT

```

```
IF (SENSE SWITCH 4) 37,57
 57  GO TO C202,203,204,1W
202  IF (V<4) - W(400)51,51,31
203  IF (W<28) - W(400)54,54,31
204  IF (W<0) - W(400)54,54,31
200  IF (XW11-59)201,59,201
201  IF (XW11-66) 37,66,37
ENDC(1,0,1,0,0,0,0,0,0,0,0,0,0,0)
```

STORAGE LOCATIONS FOR VARIABLES APPEARING IN COMMON STATEMENTS

	DEC	OCT	DEC	OCT	DEC	OCT	DEC	OCT	DEC	OCT	
G	32155	76645	N	32188	76674	RECORD	32561	77461	V	32549	77445
XN	32187	76673	YN	32180	76664	ZN	32173	76655	W	32438	77266

EXTERNAL FORMULA NUMBERS WITH CORRESPONDING INTERNAL FORMULA NUMBERS AND OCTAL LOCATIONS

	EFN	IFN	LOC											
67	4	00000	60	5	00000	110	6	00000	111	7	00000	112	8	00000
113	9	00000	114	10	00000	115	11	00000	119	12	00000	66	17	00035
66	21	00047	59	24	00066	59	25	00073	61	26	00103	62	27	00107
63	28	00170	64	29	00173	65	30	00175	100	31	00177	101	32	00200
102	33	00203	103	34	00204	104	35	00206	105	36	00207	106	37	00210
107	38	00214	107	38	00214	108	40	00220	1	41	00221	2	85	00510
2	86	00510	3	88	00520	3	89	00520	4	90	00940	5	91	00554
6	92	00557	6	93	00557	7	94	00562	7	95	00562	8	96	00566
9	97	00570	9	98	00570	10	99	00610	11	103	00621	11	104	00621
12	105	00624	12	106	00624	13	107	00630	14	108	00634	15	109	00637
16	113	00655	16	114	00655	17	115	00675	18	121	00712	19	122	00716
20	123	00722	22	124	00726	23	125	00731	24	126	00737	25	127	00745
26	128	00763	26	129	00763	28	130	00767	29	133	01005	30	134	01015
31	136	01035	32	137	01037	33	138	01041	34	139	01042	35	140	01045
36	141	01047	37	142	01054	37	142	01054	38	145	01061	39	148	01067
40	149	01071	41	150	01073	42	151	01075	43	152	01111	44	153	01101
45	154	01103	46	155	01105	47	156	01107	48	157	01111	49	158	01114
50	159	01116	53	163	01126	53	164	01126	51	165	01140	54	166	01145
57	168	01152	118	169	01155	118	169	01155	116	170	01160	56	176	01235
57	179	01242	202	179	01245	203	180	01252	204	181	01257	200	182	01265
201	183	01271												

STORAGE NOT USED BY PROGRAM

	DEC	OCT	DEC	OCT
	32196	76634		

LOCATIONS OF NAMES IN TRANSFER VECTOR

	DEC	OCT	DEC	OCT	DEC	OCT	DEC	OCT	DEC	OCT				
INT	14	00016	INTM	15	00017	EXIT	12	00014	C05	6	00006	COORD	8	00010
ARCOS	10	00012	OUTONE	13	00015	OUTPUT	16	00020	QATAN	11	00013	SIN	5	00005
SLANTR	7	00007	SORT	9	00011	(FILE)	2	00002	CFPTD	0	00000	(RTN)	4	00004
(SPHD)	1	00001	(TSH)	3	00003									

STORAGE LOCATIONS FOR VARIABLES NOT APPEARING IN DIMENSION, EQUIVALENCE OR COMMON SENTENCES

	DEC	OCT	DEC	OCT	DEC	OCT	DEC	OCT	DEC	OCT	DEC	OCT		
IN	366	01542	ID	865	01541	GAMMA	864	01540	DENM	863	01537	DATA	862	01536
ARGSE	861	01535	ARGSR	860	01534	ARGSR	859	01533	ARG5R	858	01532	ARGSR	857	01531
ARG6B	856	01530	ARG5F	855	01527	ARG5B	854	01526	ARG4M	853	01525	ARG3M	852	01524
ARG2R	851	01523	ARG2B	850	01522	ARG1R	849	01521	ARG1B	848	01520	ARG1R	847	01517
ANALPH	846	01516	K	845	01515	KWIT	844	01514	NA	843	01513	PH1B	842	01512
PHIR	841	01511	PHISP	840	01510	P1	839	01507	QB	838	01506	QR	837	01505
QSP	836	01504	RSP	835	01503	SIGH	834	01502	SIGPH1	833	01501	SIGR	832	01500
SIGTHE	831	01477	SPPS	830	01475	TE1	825	01475	TE2	826	01474	TE3	827	01473
TERM2	826	01472	TERM*	825	01471	TERMP5	824	01470	TERM6	823	01467	TERM7	822	01466
THETB	921	01465	THETR	920	01464	THETSP	919	01463	X0	918	01462	XR	917	01461

XSP	816 01460	Y0	815 01457	YR	814 01456	VSP	813 01455	20	812 01454
ZERO	811 01453	ZR	810 01452	ZSP	809 01451				

STORAGE LOCATIONS FOR SYMBOLS NOT APPEARING IN SOURCE PROGRAM

	DEC	OCT	DEC	OCT	DEC	OCT	DEC	OCT	DEC	OCT
E20J	553 01051	E>103	126 00176	E>10	696 01270	E>R	673 01241	E2N	617 01151	
E2H	548 01044	E>C	139 00213	E>9	130 00202	E6	119 00167	D601	27 00033	
D240U	692 01264	D>01	28 00034	D>10J	555 01053	C2G3	808 01450	C2G1	807 01447	
S23N	731 01333	S>3J	740 01344	S>31	763 01373	S23H	764 01374	S23G	773 01405	
S23F	783 01417	S>3E	792 01430	S>23	797 01435	S215	795 01433	S217	798 01436	
S2	702 01276	S>3	716 01314	4>	32767 77777	6>	721 01321			

ENTRY POINTS TO SUBROUTINES NOT OUTPUT FROM LIBRARY.

CPPT2	(SPH)	CFIL>	CRTND	SIN	COORD.	SORT
ARCOS	DATAH	EXIT	INT	COS		

INTW

OUTPUT

RM 61 TMP-32

00000	(FP7)	BCD 1 (FP7)	CAL 2)	SXD 62+4,4	ALS 17
00001	(SPH3	BCD 1 (SPH3)	000667	SXD 62+4,4	XCA
00002	(FILE	BCD 1 (FILE)	00070	TSX CTSHD,4	MPS 12+2
00003	(TSHD	BCD 1 (TSHD)	00071	P2E 8215	ALS 17
00004	(RTN)	BCD 1 (RTN)	00072	LXD 62+4,4	T2E ED6
00005	SIN	BCD 1 SIN	00073	STR 28A	TPL 29A
00006	COS	BCD 1 COS	00074	STR K	TRA 29A
00007	SLANTR	BCD 1 SLANTR	00075	STR	SXD CDG1,1
00010	COORD	BCD 1 COORD	00076	STO DATA	CLA DATA
00011	SQRT	BCD 1 SQRT	00077	SXD 62+4,4	FDP 3)
00012	ARCOS	BCD 1 ARCS	00100	TSX CTRNH,4	STO DATA
00013	DATA	BCD 1 DATA	00101	LXD 62+4,4	CLA DATA
00014	EXIT	BCD 1 EXIT	00102	LXD K,1	STO W+1,1
00015	OUTONE	BCD 1 OUTONE	00103	CLA K	TRA 26A
00016	INT	BCD 1 INT	00104	26A1	00175 30A
00017	INTM	BCD 1 INTM	00105	TPL 27A	00176 ED108
00020	OUTPUT	BCD 1 OUTPUT	00106	TRA ED108	SXD CCG1,1
00021	SS	CLA (FP7)	00107	27A	00177 31A
00022	STZ 4)-205	STZ 4)-205	00110	SUB 2)+8	PSE 96
00023	14A	B55	00111	STO 1)+1	00203 33A
00024	15A	TSX (SPH3),4	00112	CLA K	PSE 114
00025	16A	P2E 8)3F	00113	SUB 2)+7	00204 TRA 41A
00026	B55	TSX (FILE),4	00114	STO 1)+2	00205 PSE 98
00027	HPR 28086	TSX (SPH3),4	00115	CLA K	00206 35A
00030	B55	TSX (SPH3),4	00116	SUB 2)+6	00207 36A
00031	P2E 8)31	TSX (FILE),4	00117	STO 1)+3	PSE 114
00032	B55	TSX (FILE),4	00118	CLA K	TRA ED6
00033	D)601	LXD CDG3,4	00120	SUB 2)+5	TRA 35A
00034	D)401	LXD CG1,4	00121	STO 1)+2	00212 TRA 41A
00035	17A	CAL 2)	00122	STO 1)+4	00213 EDC
00036	SXD 62+4,4	CLA 2)	00123	CLA K	SXD CG3,2
00037	TSX CTSHD,4	CAL 2)	00124	SUB 2)+4	00214 B55
00040	P2E 8)23	TSX CTSHD,4	00125	STO 1)+5	TSX (SPH3),4
00041	LXD 62+4,4	TSX CTSHD,4	00126	CLA K	P2E 8)3E
00042	18A	LXD 22+11,1	00127	SUB 2)+3	B55
00043	19A	STR RECORD+1,1	00128	STO 1)+6	TSX (FILE),4
00044	TX1 ++,1,1	STR RECORD+1,1	00129	CLA K	B55
00045	19A1	TX1 ++,1,1	00130	SUB 2)+2	00215 B55
00046	1982	TX1 19A,1,12	00131	STO 1)+7	TSX (FILE),4
00047	21A	TX1 19A,1,12	00132	CLA K	00221 41A
00050	TSX (RTN),4	TX1 19A,1,12	00133	SUB 2)+2	00222 FSB 8-13
00051	LXD 62+4,4	TX1 19A,1,12	00134	STO 1)+7	B55
00052	CAL 2)	TX1 19A,1,12	00135	CLA K	TSX SIN,4
00053	SXD 62+4,4	TX1 19A,1,12	00136	SUB 2)+1	00224 STO ARG3H
00054	TSX CTSHD,4	TX1 19A,1,12	00137	STO 1)+8	CLA W-14
00055	P2E 8)3W	TX1 19A,1,12	00138	CLA K	B55
00056	LXD 62+4,4	TX1 19A,1,12	00139	TRA 29A	TSX COS,4
00057	23A	STR 10	00140	CLA K	00231 43A
00060	STR 10	STR 10	00141	TRA 29A	CLA W-14
00061	STR 10	STR 10	00142	CLA K	B55
00062	STR KWIT	STR KWIT	00143	TRA 29A	TSX COS,4
00063	SXD 62+4,4	STR KWIT	00144	CLA K	00236 STO ARG4H
00064	TSX (RTN),4	STR KWIT	00145	TRA 29A	CLA W-15
00065	LXD 62+4,4	STR KWIT	00146	CLA K	00237 45A
			00147	TRA 29A	B55
			00148	CLA K	TSX SIN,4
			00149	TRA 29A	00240 STO ARG5
			00150	CLA K	00241 CLA W-15
			00151	TRA 29A	00242 46A
			00152	CLA K	CLA W-14
			00153	TRA 29A	CLA W-14
			00154	TRA 29A	CLA W-14
			00155	TRA 29A	CLA W-14
			00156	TRA 29A	CLA W-14
			00157	TRA 29A	CLA W-14

00243	BSS	TSX COS,4	00330	FMP ARG1R	00422	FMP G-8
00244	STO ARGGER	00331	STO G-7	00423	STO YN-4	
00245	CLA W-16	00332	LDQ ARG3W	00424	74A	
47A	BSS	00333	FMP ARG5R	00425	FMP G-7	
00246	TSX SIN,4	00334	STO 12+1	00426	STO YN-5	
00247	STO ARG67R	00335	LDQ ARG2R	00427	75A	
00250	CLA W-16	00336	FMP ARG6R	00430	FMP G-1	
48A	BSS	00337	XCA ARG4H	00431	STO YN-6	
00251	TSX COS,4	00340	FMP ARG4M	00432	76A	
00252	STO ARG9R	00341	FAD 12+1	00433	FMP G-4	
00253	CLA W-17	00342	STO G-2	00434	STO ZN	
49A	BSS	00343	LDQ ARG4H	00435	77A	
00254	TSX SIN,4	00344	FMP ARG1R	00436	STO G-2	
00255	STO ARG9R	00345	STO G-3	00437	STO ZN-1	
00256	50A	00346	LDQ ARG3M	00440	78A	
00257	BSS	00347	FMP ARG6R	00441	FMP G-5	
00260	TSX COS,4	00350	STO 12+1	00442	STO ZN-2	
00261	STO ARG10R	00351	LDQ ARG2R	00443	79A	
51A	CLA W-62	00352	FMP ARG5R	00444	FMP G-2	
00262	UFA 65	00353	XCA	00445	STO ZN-3	
00263	LRS	00354	FMP ARG4H	00446	80A	
00264	ALA 62+1	00355	CHS	00447	FMP G-3	
00265	LLS	00356	FAD 12+1	00448	STO ZN-4	
00266	ALS 18	00357	STO G-8	00450	81A	
00267	STO IN	00358	LDQ G-4	00451	LDQ G	
00270	LXO IN,2	00359	FMP G-8	00452	FMP G-5	
00271	CLA 3D+1	00362	STO XN	00453	STO ZN-5	
52A	STO PI	00363	LDQ G-3	00454	82A	
00272	STO P1	00364	FMP G-7	00455	FMP G-2	
00273	STO ARG4M	00365	STO XN-1	00456	STO ZN-6	
53A	FMP ARG5R	00366	LDQ G-6	00457	83A	
00274	STO 12+1	00367	FMP G-5	00460	FMP ZN-5	
00275	LDQ ARG6R	00370	STO XN-2	00461	STO 12+1	
00276	FMP ARG2R	00371	LDQ G-6	00462	LDQ ZN-4	
00277	XCA	00372	FMP G-4	00463	FMP G-8	
00300	FMP ARG3M	00373	STO XN-3	00464	STO 12+2	
00301	FSS 1D+1	00374	LDQ G-3	00465	LDQ G-6	
00302	STO G	00375	FMP G-8	00466	FMP ZN-3	
00303	LDQ ARG1R	00376	STO XN-4	00467	STO 12+3	
54A	FMP ARG3M	00377	LDQ G-5	00470	LDQ ZN-2	
00305	STO G-3	00400	FMP G-7	00471	FMP G-6	
00306	LDQ ARG6R	00401	STO XN-5	00472	STO 12+4	
55A	FMP ARG4H	00402	LDQ G-18	00473	LDQ ZN-1	
00310	STO 12+1	00403	FMP G-2	00474	FMP G-7	
56A	LDQ ARG2R	00404	STO XN-6	00475	STO 12+5	
00311	FMP ARG5R	00405	LDQ G	00476	LDQ ZN	
00312	STO G-6	00412	FMP G-8	00477	FMP G-8	
00313	XCA	00406	STO YN-1	00500	FHD 12+5	
00314	FMP ARG3M	00407	LDQ G-8	00501	FHD 12+4	
00315	CHS	00410	STO YN	00502	FSB 12+3	
00316	FSS 1D+1	00411	LDQ G-7	00503	FSB 12+2	
00317	STO G-1	00412	FMP G-2	00504	FSB 12+1	
00320	LDQ ARG1R	00413	LDQ G-6	00505	STO DENM	
56A	FMP ARG6R	00414	FMP G-1	00506	85A	
00322	CHS	00415	STO YN-2	00507	STO ZN-7	
00323	STO G-1	00416	LDQ G-6	00510	86A	
00324	CLA ARG2R	00417	FMP G-2	00511	TSX SLANTR,4	
57A	STO G-4	00420	STO YN-3	00512	TSX ARG8R	
00326	LDQ ARG5R	00421	73A			
00327	SEA	00422				

RM 61 TMP-32

00513	TSX	Y-18		00600	TSX	ARG5R	00663	TSX	ARG3M
00514	TSX	Y-19		00601	TSX	ARG6R	00664	TSX	ARG4M
00515	STO	SRSP		00602	TSX	ARG7R	00665	TSX	ARG5B
00516	CLA	22+9		00603	TSX	ARG8R	00666	TSX	ARG6B
00517	STO	N		00604	TSX	ARG9R	00667	TSX	ZERO
88A	BSS	COORD.4		00605	TSX	ARG10R	00670	TSX	ZERO
00520	STO	XSP		00606	TSX	ZERO	00671	TSX	ZERO
00521	TSX	YSP		00607	TSX	Y-18	00672	TSX	ZERO
00522	TSX	ZSP		00610	CLA	ZR.	00673	TSX	ZERO
00523	TSX	ARG1R		00611	FDP	Y-18	00674	TSX	W-18
00524	TSX	ARG2R		00612	STQ	QR	00675	CLA	W-18
00525	TSX	ARG3M		00613	CLA	XR	00676	FDP	W-22
00526	TSX	ARG4M		00614	STO	W-51	00677	STO	W-29
00527	TSX	ARG5R		00615	CLA	YR	00700	CLA	Z0
00530	TSX	ARG5R		00616	STO	W-52	00701	FDP	W-18
00531	TSX	ARG6R		00617	CLA	ZR	00702	STQ	QB
00532	TSX	ARG7R		00620	STO	W-53	00703	BSS	
00533	TSX	ARG8R		00621	BSS		00703	TSX	ARCOS,4
00534	TSX	ARG9R		00622	TSX	ARCOS,4	00704	TSX	QB
00535	TSX	ARG10R		00623	TSX	OR	00705	STO	THETB
00536	TSX	SRSP		00624	TSX	QATAN,4	00706	TSX	QATAN,4
00537	TSX	Y-13		00625	TSX	XR	00707	TSX	X0
00540	Ldq	ZSP		00626	TSX	YR	00710	TSX	V0
00541	FDP	ZSP		00627	STO	PHIR	00711	CLA	Z0
00542	STO	12+1		00628	CLA	Y-18	00712	121A	CLA
00543	Ldq	YSP		00630	FDP	KSP	00713	FDP	W-18
00544	FDP	YSP		00631	FMP	ARG8R	00714	FMP	W-29
00545	STO	13+2		00632	STO	ARGSE	00715	STO	W-32
00546	Ldq	XSP		00633	Ldq	QSP	00716	122A	CLA
00547	FDP	XSP		00634	Ldq	QSP	00717	FDP	W-18
00550	FDP	12+2		00635	FMP	QSP	00720	FMP	W-29
00551	FDP	12+1		00636	STO	TERM2	00721	STO	W-30
00552	BSS	SORT,4		00637	CLA	Y-12	00722	123A	CLA
00553	STO	SRSP		00640	FSP	Y-21	00723	FDP	W-18
00554	CLA	ZSP		00641	BSS		00724	FMP	W-29
00555	FDP	RSP		00642	TSX	SIN,4	00725	STO	W-31
00556	STO	QSP		00643	STO	ARG1B	00726	124A	CLA
92A	BSS			00644	CLA	W-12	00727	FSP	PHIR
00557	TSX	ARCOS,4		00644	FSP	W-21	00730	STO	TERM4
00560	JSX	QSP		00645	BSS		00731	125A	LQ
00561	STO	TMETSP		00646	TSX	C05,4	00732	FMP	QR
00562	BSS	QATAN,4		00647	STO	ARG2B	00733	CHS	
93A	TSX	XSP		00647	CLA	W-20	00734	FAD	3D+3
00563	TSX	YSP		00650	BS5		00735	TSX	SQRT,4
00564	TSX	ZSP		00651	STO	ARG5B	00736	STO	TERMS
00565	STO	PHISP		00652	CLA	W-20	00737	126A	CLA
96A	CLA	32+2		00653	BS5		00738	BSS	TERM4
00567	STO	ZERO		00653	TSX	COS,4	00740	TSX	SIN,4
97A	BSS			00654	STO	ARG6B	00741	STO	1D+1
00570	TSX	COORD,4		00655	113A	BSS	00742	Ldq	TERME
98A	TSX	XR		00655	114A	TSX	00743	FMP	1D+1
00571	TSX	YR		00656	00743	TSX	00744	STO	TERM6
00572	TSX	ZR		00657	TSX	YO-	00745	127A	CLA
00573	TSX	ARG1R		00657	00745	TSX	00746	FSP	TERM2
00574	TSX	ARG2R		00660	00746	TSX	00747	TSX	QB/CB
00575	TSX	ARG3M		00661	00747	TSX	00747	TSX	QB/CB
00576	TSX	OCAM		00662	00747	TSX	00747	TSX	QB/CB
00577	TSX	OCAM		00663	00747	TSX	00747	TSX	QB/CB

				MSE	97	01123 162A	TSX SIGN
00750	XCA	GR	-	01035 136A	TRA 139A	01124	TSX 1D
00751	FMP	GR	-	01036 137A	CLA 32+3	01125	BSS
00752	STO	1>2	-	01040	STO SIGN	01126	TSX INT, 4
00753	CLA	TERM4	-	01041 138A	TRA 141A	01127	TSX U
055	TSX	COS, 4	-	01042 139A	MSE 98	01130	TSX N
00754	XCA	-	-	01043	TRA DD40V	01131	TSX NA
00755	FMP	DSP	-	01044 EDH	SXD CJG3, 2	01132	TSX W-41
00756	XCA	-	-	01045 140A	CLS 32+3	01133	TSX W-42
00757	FMP	TERMS	-	01046	STO SIGN	01134	TSX W-43
00760	FSB	1>2	-	01047 141A	PSE 115	01135	TSX W-44
00761	STO	TERM7	-	01050	TRA 145A	01136	TSX W-45
00762	BSS	-	-	01051 2>20J	SXD CJG3, 2	01137	TSX W-46
00763	TSX	QATAN, 4	-	01052	TRA 142A	01140	CLQ U-3
00764	TSX	TERM7	-	01053 2>10J	LXD CJG1, 1	01141	FSB W-18
00765	TSX	TERM6	-	01054 142A	BSS	01142	TZE 136A
00766	STO	ANALPH	-	01055	TSX CSPHD, 4	01143	TPL 166A
00767	LDG	ARGSE	-	01056	PZE 8>3G	01144	TRA 136A
00770	FMP	ARGSE	-	01057 143A	BSS	01145	BSS
00771	CHS	-	-	01058 44A	TSX CFILD, 4	01146	TSX INTM, 4
00772	FAD	3>3	-	01059 144A	BSS	01147	TRA EDN
00773	TSX	SORT, 4	-	01060	TSX EXIT, 4	01150	TRQ 170A
00774	STO	1>1	-	01061 145A	CLA W-48	01151	EDN SXD CG3, 2
00775	LDG	W	-	01062	STO W	01152	CLA W
00776	FMP	1>1	-	01063 146A	CLA SIGN	01153	168A TZE 169A
00777	STO	SIGR	-	01064 147A	STO W-66	01154	TPL 176A
01000	131A	CLA	3>1	01065 147A	CLA 32+2	01155	BSS TSX CSPHD, 4
01001	STO	P1	-	01066	STO W-69	01156	PZE 8>3N
01002	132A	CLA	ANALPH	01067 148A	CLA SRSP	01157	BSS TSX CFILD, 4
01003	FSB	P1	-	01068	STO V-1	01160	170A CLA W-3
01004	STO	GAMMA	-	01069	CLA V-23	01161	FSB W-18
01005	133A	CLA	GAMMA	01070 149A	STO V-2	01162	STO TE3
01006	TSX	COS, 4	-	01073 150A	CLA RSP	01163	LDQ V-4
01007	STO	1>1	-	01074 151A	STO V-3	01164	FMP 3D
01010	LDG	W	-	01075 151A	CLA THETSP	01165	STO TE
01011	FMP	ARGSE	-	01076 152A	STO V-4	01166	LDQ V-5
01012	XCA	-	-	01077 152A	CLA PHISP	01167	LDQ 172A
01013	FMP	1>1	-	01078 153A	STO V-5	01168	172A CLA SIGR
01014	STO	SIGTHE	-	01079 153A	STO V-6	01169	FMP 3D
01015	134A	CLA	GAMMA	01080 154A	CLA SIGTHE	01170	STO TE2
01022	BSS	-	-	01081 154A	STO V-7	01171	LDQ V-69
01023	FMP	1>1	-	01082 155A	CLA SIGPHI	01172	UFA 62
01024	CHS	SIN, 4	-	01083 155A	STO V-10	01173	LRS ANA 62+1
01025	STO	SIGPHI	-	01084 155A	CLA W-50	01174	PZE 8>3J
01026	135A	CLA	W-40	01085 155A	STO V-11	01175	LLS
3	01027	UFA	62	01086 155A	CLA W-49	01176	ALS 18
01030	LPS	-	-	01087 155A	STO V-1	01177	STR
01031	ANA	62+1	-	01088 156A	CLA 1D	01204	LDQ V-2
01032	LLS	-	-	01089 156A	CLA SRSP	01205	STR 22+11
01033	ALS	18	-	01090 156A	STO 1D	01206	LDQ V-1
01034	STO	NA	-	01091 161A	BSS	01207	STR

RM 61TMP-32

01210	L00 TE3	OCT +00000160000000	01372	BCD 1 INT.4H
01211	STR	OCT +00000170000000	01373	BCD 1 C5H0
01212	L00 TE1	OCT +00000200000000	01374	BCD 1 C216
01213	STR	OCT +00000210000000	01375	BCD 1 EXIT
01214	L00 TE2	OCT +00000220000000	01376	BCD 1 DV TO
01215	STR	OCT +00000250000000	01377	BCD 1 - REQ
01216	L00 V-6	OCT +00000260000000	01400	BCD 1 MONITOR
01217	STR	OCT +00000110000000	01401	BCD 1 FOR M
01220	L00 V-7	OCT +00000020000000	01402	BCD 1ITCHES
01221	STR	OCT +00000100000000	01403	BCD 1NSE SW
01222	L00 V-8	OCT +00000730000000	01404	BCD 1SET. SE
01223	STR	OCT +00000200000000	01405	BCD 1C4H0
01224	L00 W	OCT +02000000000000	01406	BCD 1CUTE)
01225	STR	OCT +206712273407	01407	BCD 1NG EXE
01226	L00 W-1	OCT +202622077326	01410	BCD IN DURI
01227	STR	OCT +00000000000000	01411	BCD 1SITI AL
01230	L00 W-27	OCT +223447600000	01412	BCD IN P
01231	STR	OCT +23300000000000	01413	BCD 1CHES 1
01232	L00 W-65	OCT +00000007777777	01414	BCD 1E SWIT
01233	STR	OCT +00000000000000	01415	BCD 1L SENS
01234	BSS	OCT +00000100000000	01416	BCD 1SET AL
01234	TSX CFIL),♦	OCT +00000000000000	01417	BCD 1C54H0
01235	TSX OUTPUT,♦	OCT +02000000000000	01420	BCD 1RED)
01236	PSE 116	OCT 1 OR ZE	01421	BCD 1NSIDE
01237	TRA EDR	OCT 1GATIVE	01422	BCD 1EING C
01240	TRA 142A	OCT 1 IS	01423	BCD 1BLEM B
01241	EDR SXO CG3.2	OCT 1 W(1)	01424	BCD 1OR PRO
01242	TRA 178A	OCT 1C28H0	01425	BCD 1SS2 F
01243	TRA 181A	OCT 1E9-22	01426	BCD 1SL AND
01244	TRA 180A	OCT 1,1P3	01427	BCD 1 SET S
01245	CLA V-3	OCT 1H ,1PE	01430	BCD 1 C46H0
01246	FSB W-38	OCT 1E9-2,1	01432	BCD 1E14.7
01247	TZE 179A1	OCT 1,1P	01433	BCD 1 C13,
01250	TPL 136A	OCT 19-2,2H	01434	BCD 12
01251	TRA 165A	OCT 19-2,3H	01435	BCD 1 C12A6
01252	CLA W-27	OCT 15,1P7E	01435	BCD 1 C10,1
01253	FSB W-39	OCT 1C10,1	01435	BCD 1XJ
01254	180A1	OCT 1346	BCD 1TX,1H	
01255	TPL 136A	OCT 1347	BCD 1HRANGE	
01256	TRA 166A	OCT 1350	BCD 12,5X,5	
01257	181A	OCT 1351	BCD 1,4HW,C2	
01260	CLA W-39	OCT 1352	BCD 1C12,6X	
01261	TZE 166A	OCT 1353	BCD 15X,4H	
01262	TPL 136A	OCT 1354	BCD 13PHM1,	
01263	TRA 166A	OCT 1355	BCD 16X,6H5	
01264	LxD CG1,4	OCT 1356	BCD 1THETA,	
01265	CLA KWT	OCT 1357	BCD 1,8HSIG	
01266	SUB 20+12	OCT 1360	BCD 1IGR,3X	
01267	TZE 24A	OCT 1361	BCD 15X,4H5	
01270	SXD CG3,2	OCT 1362	BCD 13PHM1,	
01271	CLA KWT	OCT 1363	BCD 1TA,5X,	
01272	SUB 20+13	OCT 1364	BCD 1,5HTHE	
01273	TZE 17A	OCT 1365	BCD 1-R,3X	
01274	TPL D10J	OCT 1366	BCD 17HVC4	
01275	TRA D10J	OCT 1367	BCD 12),4X,	
01276	2>	OCT +000005000000	BCD 1X,4HVC	
01277	OCT +000015000000	OCT 1371	BCD 1DELV,5	

B. FUNCTION SLANTR

This function is used to calculate the non-deviated ray path between the transmitter, R, and the starting point, S, (See Figures 1 and 2) from the given input data describing the problem under consideration. The input data should be designed in a manner that this assumption is true.

```
C  FUNCTION SLANTR, DECEMBER 22, 1960, IBM-7090
1  FUNCTION SLANTR(ARG1,ARG2,ARG13)
2  TERM12 = SQRT((C1.0+ CARG13*GR312)**2 - ARG3**2)
3  SLANTR = ARG12* C-ARG37* TERM12
4  RETURN
5  END(0,1,0,0,0,0,0,0,0,0,0,0)
```

EXTERNAL FORMULA NUMBERS WITH CORRESPONDING INTERNAL FORMULA NUMBERS AND OCTAL LOCATIONS

	EFN ₁	IFN ₁	LOC ₁	EFN ₂	IFN ₂	LOC ₂	EFN ₃	IFN ₃	LOC ₃
	1, 00000			2, 00022			3, 4, 00049		

STORAGE NOT USED BY PROGRAM

	DEC	OCT
	55, 00057	32551, 77461

LOCATIONS OF NAMES IN TRANSFER VECTOR

	DEC	OCT	DEC	OCT	DEC	OCT
	327, 0, 00000					

STORAGE LOCATIONS FOR VARIABLES NOT APPEARING IN DIMENSION, EQUIVALENCE OR COMMON SENTENCES

	DEC	OCT	DEC	CST	DEC	OCT	DEC	OCT
	54, 00066	20, ...	53, 00065		52, 00065		51, 00065	

STORAGE LOCATIONS FOR SYMBOLS NOT APPEARING IN SOURCE PROGRAM

	DEC	OCT	DEC	OCT	DEC	OCT	DEC	OCT
10	50, 00062	20, ...	43, 0, 053	32, ...	44, 00054	62, ...	45, 00055	

ENTRY POINTS TO SUBROUTINES NOT OUTPUT FROM LIBRARY.

SORT

RM 61TMP-32

00000	32P1	BLQ 150RT
00001	3	HTR
00002		HTR
00003		HTR
00004*		BLQ ISLANTR
00005		SXO 5,1
00006		SXO 3+1,2
00007		SXO 3+2,4
00008		CLA 1,2
00009		STO 3R+14
00010		CLA 1,2
00011		STO 3R+14
00012		CLA 1,2
00013		STO 3R 3R+1
00014		CLA 3,4
00015		STO 3R+2
00016		STO 3R+17
00017		CLA 3R+17
00018		CLA 3,4
00019		STO 3R+3
00020		CLA 3R+3
00021		STO 3R+3
00022	32	DQ ARG8
00023		FMP ARG8
00024		STO 12,1
00025		CLA ARG13
00026		FOP ARG12
00027		STO 12+2
00028		CLA 32
00029		FAD 12+2
00030		STO 12
00031		LQ 12
00032		FOR 12
00033		F5B 12+1
00034		ESS
00035		TSX SORT,4
00036		TSX JERB1
00037		CLS ARG7
00038	4A	FAD JERB1
00039		STO 12+1
00040		LQD ARG12
00041		FMP 12+1
00042		STO 12+1
00043		LQD ARG12
00044		FMP 12+1
00045		STO SLANTR
00046	5A	CLA SLANTR
00047		LQD 5,1
00048		LQD 5+1,2
00051		LQD 5+2,4
00052		TRA 5,4
00053	22	OCT *0000002002000000
00054	32	OCT *00140000000000
00055	62	OCT *23300000000000
00056		OCT *00000000777777
00057		OCT *00000000000000
00060		GCT *000001000000
00061		OCT *00000000000000

C. FUNCTION QATAN

Function QATAN permits the evaluation of the value of the arctangent of an angle with proper quadrant allocation.

RM 61 TMP-32

```

FUNCTION DATAN, DECEMBER 22, 1961 - 161-7020
      PT = 3.1415927
      IF(COSFCOD-1.0E-18D9.2,2
      DATAN = PI/2.0
      GO TO 6
      RETURN = Y/A
      IF(COS3D 4,13,5
      AS3 = -925
      DATAN = ATAN(COS3D)
      IF(COD 8,7,7
      IF(COD-10.5,8,18-
      IF(COD 12,11,11
      DATAN = C2.0*PI/2.0-DATAN
      GO TO 19
      GO TO 14 = PI - DATAN
      GO TO 18
      DATAN = PI + DATAN
      GO TO 19
      IF(COD 14,15,16
      DATAN = PI
      GO TO 18
      DATAN = 0.0
      RETURN
      ENDG, 1.0E7,0,0,0,0,0,0,0,0,0
    )
  )

```

EXTERNAL FORMULA NUMBERS WITH CORRESPONDING INTERNAL FORMULA NUMBERS AND OCTAL LOCATIONS

	IFN	LOC								
1	00000	9	00031	17	00037	2	00043	3	00046	
2	00051	5	00053	6	00056	7	00062	8	00066	
3	00012	11	00077	12	00103	13	00107	14	00112	
4	00114	15	00115	16	00117	18	00117	19	00117	

STORAGE NOT USED BY PROGRAM

	DEC	OCT	DEC	OCT	DEC	OCT	DEC	OCT	DEC	OCT
38	00142		32561	7761						

LOCATIONS OF NAMES IN TRANSFER VECTOR

	DEC	OCT	DEC	OCT	DEC	OCT	DEC	OCT	DEC	OCT
ATAN	-35	757	00000							

STORAGE LOCATIONS FOR VARIABLES NOT APPEARING IN DIMENSION, EQUIVALENCE OR COMMON SENTENCES

	DEC	OCT	DEC	OCT	DEC	OCT	DEC	OCT	DEC	OCT
ARG	37	00141	P1	36,00140	34ATAN	35,00137				

STORAGE LOCATIONS FOR SYMBOLS NOT appearing IN SOURCE PROGRAM

	DEC	OCT	DEC	OCT	DEC	OCT	DEC	OCT	DEC	OCT
40	34,00124	35	34,00125	65	34,00132	90	34,00132	90		

ENTRY POINTS TO SUBROUTINES NOT OUTPUT FROM LIBRARY.

ATAN

RM 61TMP-32

00000 ATAH	BCD JATAN	06071	TBA 19A
00001 S	HTR	06072	L08 30+3
00002	HTR	06073	FMP PI
00003	HTR	06074	FSB QATAN
00004	BCD JATAN	06075	STO QATAN
00005	SXO S.1	06076	TBA 25A
00006	BCD S.2	06077	CLA PI
00007	SXO S.2	06078	FSB QATAN
00008	CLA J.4	06100	STO QATAN
00009	STA J.4	06101	STO QATAN
00010	CLA 2.4	06102	TBA 25A
00011	STA 3A.2	06103	CLA PI
00012	STA 3A.8	06104	FAD QATAN
00013	STA 3A.8	06105	STO QATAN
00014	STA 3B.33	06106	TBA 25A
00015	CLA 2.4	06107	CLA X
00016	STA 3B.7	06108	TZC 24A
00017	STA 3A.17	06110	CLA 24A
00018	STA 3B.32	06111	TPL 24A
00019	STA 3B.36	06112	CLA PI
00020	CLA 32	06113	STO QATAN
00021	STA P1	06114	TBA 25A
00022	CLA X	06115	CLA 32+4
00023	STO P1	06116	STO QATAN
00024	SSP	06117	CLA 32+4
00025	SSP	06118	CLA 32+4
00026	FSP 32+2	06119	CLA 32+4
00027	TZC 36	06120	L00 S.1
00028	TPL 88	06121	L00 S.1+2
00029	CLA Y	06122	L00 S.2+4
00030	SZP	06123	TBA 3A
00031	FSP 35+2	06124	OCT +0000000000000000
00032	TZC 88	06125	OCT +2300000000000000
00033	CLA Y	06126	OCT +105447113564
00034	FSP 35+2	06127	OCT +224674055331
00035	TPL 68	06128	OCT +20240000000000
00036	IRB 88	06129	OCT +0000000000000000
00037	CLA P1	06130	OCT +0000000000000000
00038	FDP 32+3	06131	OCT +0000000000000000
00039	STO QATAN	06132	OCT +2300000000000000
00040	STO QATAN	06133	OCT +0000000000000000
00041	TBA 12A	06134	OCT +0000000000000000
00042	CLA Y	06135	OCT +0000000000000000
00043	FDP X	06136	OCT +0000000000000000
00044	STO ARG	06137	OCT +0000000000000000
00045	STO ARG	06138	OCT +0000000000000000
00046	CLA ARG	06139	OCT +0000000000000000
00047	TZC 21A	06140	OCT +0000000000000000
00048	TPL 11A	06141	OCT +0000000000000000
00049	CLS ARG	06142	OCT +0000000000000000
00050	STO ARG	06143	OCT +0000000000000000
00051	IRB 14A	06144	OCT +0000000000000000
00052	CLA Y	06145	OCT +0000000000000000
00053	CLA Y	06146	OCT +0000000000000000
00054	TSP ATAN	06147	OCT +0000000000000000
00055	STO QATAN	06148	OCT +0000000000000000
00056	CLA X	06149	OCT +0000000000000000
00057	TZC 12A	06150	OCT +0000000000000000
00058	TPL 13A	06151	OCT +0000000000000000
00059	IRB 14A	06152	OCT +0000000000000000
00060	CLA Y	06153	OCT +0000000000000000
00061	IRB 14A	06154	OCT +0000000000000000
00062	CLA Y	06155	OCT +0000000000000000
00063	TSP ATAN	06156	OCT +0000000000000000
00064	STO QATAN	06157	OCT +0000000000000000
00065	TPL 13A	06158	OCT +0000000000000000
00066	IRB 14A	06159	OCT +0000000000000000
00067	CLA Y	06160	OCT +0000000000000000
00068	TPL 17A	06161	OCT +0000000000000000

D. FUNCTION ARCos

Function ARCos permits the evaluation of the value of the arccosine of an angle. Presently no quadrant allocation is made.

```

C FUNCTION ARCCOS DECEMBER 22, 1961 — 13-1-7030
1      FUNCTION ARCCOS(X)
2      IF X>=0.0 AND X<=1.0E-16 RETURN 0.0
3      ARCCOS = 1.5707963
4      GO TO 8
5      B1 = Q+2
6      B2 = 3.0*PI*(1.0/Q) - 71.92
7      ARCCOS = ATAN(F(B2))
8      IF Q<0.7 AND Q>0.9
9      ARCCOS = 3.1415927 - ARCCOS
10     RETURN
11    END(0,1.0,0.0,0.0,0.0,0.0,0.0)

```

EXTERNAL FORMULA NUMBER → INTRINSIC FORMULA NUMBERS AND OCTAL LOCATIONS

	EFN	IFN	LOC	EFN	IFN	LOC	EFN	IFN	LOC
1	3	00016	• 2	4	00023	• 3	00026	4	7 00031
2	9	00042	7	10	00145	8	11	00050	5

STORAGE NOT USED BY PROGRAM

	DEC	OCT									
38	00072	33561	77491	33561	77491	33561	77491	33561	77491	33561	77491

LOCATIONS OF NAMES IN TRANSFER VECTOR

	DEC	OCT								
STAN	100001	300001	100001	300001	100001	300001	100001	300001	100001	300001

STORAGE LOCATIONS FOR VARIABLES NOT APPEARING IN DIMENSION, EQUIVALENCE OR COMMON SENTENCES

	DEC	OCT								
B1	57 00071	101 00071	56 00070	100 00070	55 00069	100 00069	54 00068	100 00068	53 00067	100 00067

STORAGE LOCATIONS FOR SYMBOLS NOT APPEARING IN SOURCE PROGRAM

	DEC	OCT								
25	45 00059	35 00059	46 00056	65 00056	46 00056	65 00056	50 00062	50 00062	50 00062	50 00062

ENTRY POINTS TO SUBROUTINES NOT CUTOFF FROM LIBRARY.

	STAN									
1	1	1	1	1	1	1	1	1	1	1

RM 61 TMP-32

00000	SEPT	B60	150RT	
00001	ATAN	B60	150TAN	
00002	5	HTR		
00003		HTR		
00004		HTR		
00005		B60	1ARCOS	
00006		SXD	3.1	
00007		SXD	3+1.2	
00010		SXD	3+2.4	
00011		CLA	3.4	
00012		STA	3.2	
00013		STA	3+3	
00014		STA	36+9	
00015		STA	34+20	
00016		CLA	2	
00017		SSP		
00018		F5B	30	
00019		TBL	45	
00020		CNA	35+1	
00021		STO	37.05	
00022		TBL	7.15	
00023		CNA	35	
00024		F5B	3	
00025		STO	81	
00026		CNA	35+2	
00027		CLB	81	
00028		F5B	81	
00029		FCB		
00030		F5B	32+2	
00031		FCB		
00032		F5B	32	
00033		FCB		
00034		F5B	32+2	
00035		FCB		
00036		F5B	32	
00037		FCB		
00038		F5B	32	
00039		FCB		
00040		T5A	ATAN, 4	
00041		STO	ARCOS	
00042	92	CLB	2	
00043	961	T2E	118	
00044		TBL	119	
00045	134	CLB	32+3	
00046		F5B	ARCOS	
00047		STO	ARCOS	
00048	113	CLB	ARCOS	
00049		LXO	3.1	
00050		LXO	3.1	
00051		LXO	3.1	
00052		LXO	3.1; 2	
00053		LXO	5+2.4	
00054		LXO	5+2.4	
00055	29	FCI	+0000000000000	
00056	32	FCI	+113713162+3	
00057		FCI	+201622077323	
00058		FCI	+20142000002022	
00059		FCI	+201622077326	
00060		FCI	+201622077326	
00061		FCI	+201622077326	
00062	62	FCI	+23300000000000	
00063		FCI	+000000077777	
00064		FCI	+00000000000000	
00065		FCI	+000001000000	
00066		FCI	+00000000000000	

E. SUBROUTINE COORD

This subroutine is used to transform the problem from the radar system to an earth centered geomagnetic spherical coordinate system.

EXTERNAL FORMULA NUMBERS WITH CORRESPONDING INTERNAL FORMA & NUMBERS AND OCTAL LOCATIONS

EFN	IFN	OCT	DEC	IFN	LOC	EFN	IFN	LOC	EFN	IFN	LOC		
1	00000	00000	2	00123	07123	3	00200	4	00240	5	00240	6	00316

STORAGE NOT USED BY PROGRAM

DEC	OCT	DEC	OCT		
224	00340	3	761	77	61

STORAGE LOCATIONS FOR SYMBOLS NOT APPEARING IN SOURCE PROGRAM

DEC	OCT	DEC	OCT	DEC	OCT	DEC	OCT
1	00330	2	0322	62	0322	211	00323

RM 61TMP-32

00296 00257 00260 00261 00262 00263 00264 00265 00266 00267 00270 00271 00272 00273 00274 00275 00277 00278 00280 00281 00282 00283 00284 00285 00286 00287 00288 00289 00290 00291 00292 00293 00294 00295 00296 00297 00298 00299 00300 00301 00302 00303 00304 00305 00306 00307 00308 00309 00310 00311 00312 00313 00314 00315 00316 00317 00318 00319 00320 00321 00322 00323 00324 00325 00326 00327

10+4
LDD RFG5
FMP ARG2
XCR
FMP ARG4
CHS
FAD 10+4
STO 10+5
LDD RFG10
PNP 10+5
XCR
FMP ARG11
XCR
FMP ARG3
STO 10+6
LDD ARG39
PNP ARG31
XCR
FMP ARG4
XCR
FMP ARG11
XCR
FMP ARG38
STO 10+7
LDD ARG37
FMP 10+2
XCR
FMP ARG11
FHC 10+7
FAD 10+6
FAD 10+3
STO 2
XCR
FMP ARG11
FHC 10+7
FAD 10+6
FAD 10+3
STO 2
XCR
LDD 3+1
LDD 3+1+2
LDD 5+2+4
TR4 16+4
OCT +0000000000000
OCT +2330000000000
OCT +00000007777
OCT +000000000000
OCT +2000001000000
OCT +000000-00000

F. SUBROUTINE DAUX

Subroutine DAUX is used to define the differential equations that are to be numerically integrated. As a result the previously described ray trace equations are defined in this subroutine.

```

C ____ SUBROUTINE DAUX DECEMBER 22-1960 ____ IBM-7090
1 ____ SUBROUTINE DAUX
DIMENSION RECORD(12), NC1110, N 25,2
COMMAND RECORD, U, N
CALL FOPEN
      TER1 = UC112 * UC113
      TER2 = UC114 * UC115
      TER3 = UC116 * UC117
      IF CABS(TER2-1.0E-32) > 1.7E-3
      TER2 = 1.0E-3
      UC112 = C1.0/TER12 * UC117 - UC113*UC115
      UC113 = C1.0/UC114 * TER12 * TER12 - UC114*UC116
      UC112 = C1.0/NC113 * UC117 + UC114 * UC115
      UC113 = C1.0/NC112 * UC112 + UC114 * UC116
      UC112 = C1.0/UC114 * UC113 * UC112 + UC115 * UC116
      UC113 = C1.0/UC115 * UC112 * UC114 + UC116 * UC117
      UC112 = C1.0/TER3 * TER3
      UC113 = C1.0/UC114 * UC112 * TER3
      UC112 = C1.0/UC113 * UC114 * TER3
      UC113 = C1.0/UC112 * UC113 * TER3
      UC112 = C1.0/UC114 * UC113 * UC114
      UC113 = C1.0/UC115 * UC114 * UC115
      UC112 = C1.0/UC116 * UC115 * UC116
      UC113 = C1.0/UC117 * UC116 * UC117
      RETURN
END

```

STORAGE LOCATIONS FOR VARIABLES APPEARING IN COMMON STATEMENTS

EXTERNAL FORMULA NUMBERS 774 CORRESPONDING INTERNAL FORMULA NUMBERS AND OCTAL LOCATIONS

	IFN	LOC	EFN	IFN	LOC	EFN	IFN	LOC	EFN	IFN	LOC
1	00000	1	00014	5	00017	6	00022	7	00026	8	00030
2	00032	7	00034	9	00045	10	00061	11	00077	12	00083
3	00033	3	00035	14	00172	15	00203	16	00213	17	00214
4	00034	15	00115	12	00141	13	00224	14	00225	15	00226

KODAK SAFETY FILM FOR 35MM CINEMA

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DEC OCT DEC CCT
1159 20351 32187 76573

LOCATIONS OF NAMES IN TRANSFER VECTOR

SEC OCT 2000C RINDEX DEC 000000 RCT SIN 1 C0001 DEC OCT 3 00003 SEC OCT 2000C RINDEX DEC 000000 RCT SIN 1 C0001 DEC OCT 3 00003

IN DIMENSION, EQUIVALENCE OR COMMON SENTENCE

TER1 DÉC OCT TER2 DÉC OCT TÉR3 DÉC OCT TER4 DÉC OCT

STORAGE LOCATIONS FOR SYMBOLS NOT APPEARING IN SOURCE PROGRAM

DEC 16 1961 00241 20 DEC 152 00230 35 DEC 153 00231 60 DEC 156 00234

ENTRY POINTS TO SUBROUTINES. NOT CUTPUT FROM LIBRARY.

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00000_RINDEX_ACO_18INDEX		00067	FDP Y-3		00161	FDP Y-3	
00001 SIN	BCD 15IN	00070	XCA		00162	STO 12+3	
00002 COS	BCD J.05	00071	FDP TER1		00163	LDD Y-3	
00003 SQR	BCD 15QRT	00072	STO 12+2		00164	FMP TER2	
00004 S	HTR	00073	CLR 32+1		00165	STO 12+4	
00005	HTR	00074	FDP 12+2		00166	CLA 32+1	
00006	HTR	00075	FDP 12+1		00167	FDP 12+4	
00007	BCD 10AUX	00076	STO U-14		00170	FDP 12+3	
00008	SIN S.1	00077	LDD TER2		00171	STD Y-7	
00009	SIN S+1.2	00100	FDP U-8		00172	LDD Y-9	
00010	SIN S+2.4	00101	XCA		00173	FMP Y-9	
00011	SIN S+2.4	00102	FDP U-14		00174	FAD TER1	
00012	SIN S+2.4	00103	STO 12+1			BSS	
00013	TSX RINDEX.4	00104	LDD U-7		00175	TSX SQRT.4	
00014	6A	00105	FDP U-13		00176	STD 12+1	
00015	FDP Y	00106	STO 12+2		00177	CLA 32+1	
00016	STO TER1	00107	CLB 32+1			FDP TER1	
00017	7A	00108	FDP U		00201	FDP 12+1	
00018	BSS	00110	STO 12+1		00202	STD Y-18	
00019	TSX SIN.4	00111	FDP Y-6				
00020	STO TER2	00112	FAD 12+2		00203	18A	CLA Y-2
00021	CLB Y-9	00113	FAD 12+1		00204	FDP Y	
00022	8A	00114	STO U-15		00205	FMP Y-10	
00023	TSX COS.4	00115	LDD JER3		00206	FAD 32+1	
00024	STO TER3	00116	FDP U-3		00207	STD 12+1	
00025	CLB TER2	00117	XCA		00211	FDP 32+2	
00026	SSP	00120	FDP U-8		00212	FMP 12+1	
00027	FDP 32	00121	XCA				
00028	TZE 11A	00122	FDP U-14		00213	STD Y-19	
00029	TPL 11B	00123	STO 12+1		00214	19B	CLA Y-1
00030	CLB 32	00124	LDD U-7		00215	FDP Y	
00031	STO JER2	00125	FDP U-12		00216	STD 12+1	
00032	10A	00126	STO 12+2		00217	CLA Y-2	
00033	LDD W	00127	CLB 32+1		00220	FDP 32+2	
00034	11A	00128	FDP U		00221	FMP 12+1	
00035	FDP N-3	00129	STO 12+1		00222	CHS	
00036	CHS	00130	00130		00223	STD Y-20	
00037	FDP Y-6	00131	FDP Y-7				
00038	12A	00132	FDP 12+2				
00039	STO 12+1	00133	FDP 12+1		00224	LDD S.1	
00040	CLB 32+1	00134	STO 12+3		00225	LDD S.1.2	
00041	FDP TER1	00135	CLB 32+1		00226	LDD S+2.4	
00042	FDP JER1	00136	FDP U-3		00227	TRA 1.4	
00043	FDP JER1	00137	FDP 12+3		00230	22	0CI +00000020000000
00044	STO Y-12	00138	FDP U-16		00231	32	0CI +14652461670
00045	12A	00139	STO U-16		00232	0CI +20146444760000	
00046	FDP Y-4	00140	LDD JER3		00233	0CI +223444760000	
00047	CLB 32+1	00141	00142		00234	0CI +23300000000000	
00048	FAD V-7	00142	FDP U-3		00235	0CI +0000000077777	
00049	STO 12+1	00143	XCA		00236	0CI +00000000000000	
00050	LDD V-3	00144	FDP U-8		00237	0CI +00000000000000	
00051	STO 12+1	00145	XCA		00240	0CI +00000000000000	
00052	LDD V-3	00146	FDP U-13				
00053	FDP TER1	00147	STO 12+1				
00054	STO 12+2	00148	LDD TER2				
00055	CLB 32+1	00149	FDP Y-8				
00056	FDP 12+2	00150	LDD TER2				
00057	FDP 12+1	00151	FDP Y-8				
00058	STO V-13	00152	XCA				
00059	JER1	00153	FDP Y-12				
00060	FDP V-5	00154	STO 12+2				
00061	CMS	00155	CLA 32+1				
00062	FAD V-8	00156	FDP Y				
00063	STO 12+1	00157	FDP Y-8				
00064	LDD TER2	00160	FSS 12+2				

G. SUBROUTINES INT AND INTM

This is a generally available SHARE program which permits the numerical integration of a chosen set of first order non-linear differential equations. It can be operated in three possible numerical integration modes (a) Runge-Kutta with a fixed integration mesh size, (b) fourth order Adams-Moulton with a fixed integration mesh size, and (c) fourth order Adams-Moulton with a variable integration mesh size that is controlled by an error sensing routine. Because no FORTRAN source program listing is available, a SHARE description of this routine is given along with a SAP listing.

IDENTIFICATION

RW INT. Adams-Moulton, Runge-Kutta Integration
704 - FORTRAN SAP Language Subroutine
Space Technology Laboratories,
Robert Causey and Werner L. Frank,
November 30, 1958

ABSTRACT

FORTRAN version of RW-DE2F which integrates a system of N simultaneous, first order, ordinary differential equations. Option of using either 4th order Runge-Kutta method or 4th order predictor-corrector method (Adams-Moulton) is provided. Also option of automatic error control with variable step-size is provided. Input and output are single precision but double precision is used internally to control round-off errors. Requires $12N + 3$ cells for data and 693 words for program.

PURPOSE

This FORTRAN subprogram integrates a set of N simultaneous, first order differential equations. It is the FORTRAN version of the standard subroutine RW-DE2F.

RESTRICTIONS

This program has two distinct entries, one for set up and the second for performing the integration steps. The user must supply a FORTRAN subprogram (with the name DAUX) which evaluates the derivatives y' .

METHOD

The user has the option of using either a fourth order Runge-Kutta method or the fourth order Adams-Moulton method with a fixed step-size. There is also a variable step-size mode.

While input and output to this routine are single precision, double precision is used internally to control round-off errors. Truncation error is controlled either by choosing an appropriate step-size, or by using the variable step-size mode of operation.

For details of the method see RW-DE2F.

USAGE

a. Calling Sequence for set up (performed prior to initiating the integration).

CALL INT (V, N, A1, A2, A3, A4, A5, A6, A7)

Where V is a region of at least dimension $12N + 3$

N is the number of equations

A1 is the option word

A2 is E

A3 is M

A4 is A

A5 is h_{\max}

A6 is h_{\min}

A7 is β

For meaning of A1 - A7 see Appendix A and B of RW-DE2F.

Region V contains the following information prior to Set Up entry.

V(2) = x, initial value of independent variable

V(3) = h, value of step-size

V(4) = y_1 }
 : } values of dependent variables y_i
V(3+N) = y_N }

$$\left. \begin{array}{l} V(4+N) = y'_1 \\ \vdots \\ V(3+2N) = y'_N \end{array} \right\} \text{values of the derivatives } y'_i \text{ to be supplied by the auxiliary DAUX.}$$

Note: This region and the parameter N should be placed in COMMON since it is necessarily referred to in the main program and in the auxiliary. The cell V (1) is set up by the subprogram RW INT and will contain N scaled at 35.

b. Calling Sequence for integrating one step.

CALL INTM

No arguments are required for this statement.

SPACE REQUIRED

693 cells

CHECKOUT

This routine has been extensively tested on several check problems. In all cases the errors were approximately equal to their expected values, and there were no indications that round-off errors accumulate rapidly.

METHOD

References:

1. S. D. Conte and J. Titus, An interpretive floating point subroutine for the solution of systems of ordinary differential equations, Appendix I, Proc. Math. Committee of Univac Scientific Exchange Meeting, Nov. 21-22, 1957 (Obtainable from Remington Rand Univac, St. Paul, Minnesota).
2. E. K. Blum, A modification of the Runge-Kutta fourth-order method, Appendix H, Proc. Math. Committee of Univac Scientific Exchange Meeting, Nov. 21-22, 1957.

In this routine the user is allowed an option of using either the Runge-Kutta classical fourth-order method as modified by E. K. Blum [Ref. (2)] or the Adams-Moulton predictor-corrector method using the Runge-Kutta method for starting the process. Let the system of equations to be solved be given in the form

$$(1) \quad \left. \begin{array}{l} y'_i = f_i(x, y_1, y_2, \dots, y_N) \\ y_i(x_0) = y_{i0} \end{array} \right\} \quad i = 1, 2, \dots, N.$$

Let y_{in} be the value of y_i at $x = x_n$ and f_i the derivation of y_i at $x = x_n$ and let h be the increment (step-size) of the independent variable x . The classical Runge-Kutta fourth-order method uses the formulas

$$\begin{aligned} k_{i1} &= h f_i(x_n, y_{in}), \\ k_{i2} &= h f_i\left(x_n + \frac{1}{2}h, y_{in} + \frac{1}{2}k_{i1}\right), \\ k_{i3} &= h f_i\left(x_n + \frac{1}{2}h, y_{in} + \frac{1}{2}k_{i2}\right), \end{aligned}$$

$$(2) \quad \begin{aligned} k_{14} &= h f_i(x_n + h, y_{in} + k_{13}), \\ y_{i,n+1} &= y_n + \frac{1}{6} (k_{i1} + 2k_{i2} + 2k_{i3} + k_{i4}), \end{aligned}$$

The following formulas (we omit the subscript i for notational simplicity) were derived by E. K. Blum to control the growth of round-off errors.

$$(3) \quad \begin{cases} z_0 = y_n, \\ q_0 = q_{4n} \end{cases}$$

$$(4) \quad \begin{cases} P_0 = h f(x_n, z_0), \\ r_1 = L^{(1)} R^{(1)} \left[\frac{1}{2} P_0 - q_0 \right], \\ z_1 = z_0 + r_1, \\ q_1 = 3r_1 - \left[\frac{1}{2} P_0 - q_0 \right], \end{cases}$$

$$(5) \quad \begin{cases} P_1 = h f(x_n + \frac{1}{2}h, z_1), \\ r_2 = L^{(2)} R^{(2)} \left[\frac{1}{2} P_1 - \frac{1}{2} q_1 \right], \\ z_2 = z_1 + r_2, \\ q_2 = -r_2 - \frac{1}{3} q_1 + \frac{1}{2} P_1, \end{cases}$$

$$(6) \quad \begin{cases} P_2 = h f(x_n + \frac{1}{2}h, z_2), \\ r_3 = L^{(3)} R^{(3)} [P_2], \\ z_3 = z_2 + r_3, \\ q_3 = -r_3 + q_2, \end{cases}$$

$$(7) \quad \begin{cases} P_3 &= h f(x_n + h, z_3) + 2P_2 , \\ r_4 &= L^{(4)} R^{(4)} \left[\frac{1}{6} P_3 + q_3 \right] , \\ y_{n+1} &= z_4 = z_3 + r_4 , \\ q_{4,n+1} &= 3 \left[r_4 - \left(\frac{1}{6} P_3 + q_3 \right) \right] , \end{cases}$$

where $R^{(m)}$, $L^{(m)}$ denote operators which shift right m places or left m places respectively and q_{40} is taken to be zero to start the computation. (See Ref (2) for a complete description of this method.) Formulas (3) - (7) are those used in this routine.

The Adams-Moulton predictor-corrector formulas for the system (1) are

$$(8) \quad y_{i,n+1}^{(p)} = y_{in} + \frac{h}{24} (55f_{in} - 59f_{i,n-1} + 37f_{i,n-2} - 9f_{i,n-3}) ,$$

$$(9) \quad y_{i,n+1}^{(c)} = y_{in} + \frac{h}{24} (9f_{i,n+1}^{(p)} + 19f_{in} - 5f_{i,n-1} + f_{i,n-2}) .$$

The corrector formula (9) is applied only once so that only two derivative evaluations are needed for each Adams-Moulton integration step. The starting values needed in (8) are obtained using the Runge-Kutta-Blum (RKB) method.

The Adams-Moulton method may be used either with a fixed step-size or with a variable step-size. The step-size to be used in the variable mode is determined as follows. Let

$$(10) \quad \begin{aligned} E_{n+1} &= \text{Max}_i \left| \frac{y_{i,n+1}^{(p)} - y_{i,n+1}^{(c)}}{14D_i} \right| , \\ D_i &= \text{Max} \left\{ \left| y_{i,n+1}^{(c)} \right| , A \right\} , \end{aligned}$$

where $A > 0$. The user will specify an upper bound \bar{E} on the truncation error estimate E_{n+1} . This is equivalent to specifying the number of significant figures which the user desires to preserve locally throughout the integration. There must also be specified a constant $M > 0$

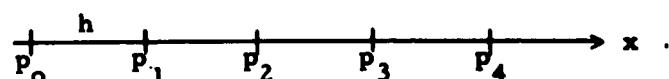
from which a lower bound $\underline{E} = M^{-1} \bar{E}$ is obtained. M should normally range from 50 to 150. The interval will then be decreased, left as it is, or increased according as the following inequalities hold:

- (11a) If $E_{n+1} \geq \bar{E}$, the interval is reduced to βh ($0 < \beta < 1$)
- (11b) If $\underline{E} \leq E_{n+1} < \bar{E}$, the interval size is kept fixed.
- (11c) If $E_{n+1} < \underline{E}$ for 3 successive steps, the step-size is increased to $\frac{1}{\beta} h$.

Normally, the routine will take $\beta = 1/2$, unless β is otherwise specified. The constant A in (10) is used to prevent unnecessary reductions in $|h|$ whenever $|y_{i,n+1}|$ is small. Normally the routine will set $A = 1$. However, some other value for A may be specified by the user if he desires to use some other characteristic length for A . While the test based on (10) will guarantee that the local error does not exceed \bar{E} , the cumulative error will usually exceed \bar{E} . Hence, \bar{E} should be chosen small enough to allow for an accumulation of truncation error. Normally \bar{E} should be in the range $10^{-8} < \bar{E} < 10^{-3}$.

After an interval is increased, the program prevents increasing again until 6 more points have been completed. However, the program may decrease the interval as often as necessary.

Starting values for the Adams-Moulton formulas are always obtained using the RKB method whenever the interval size is changed, just as at the beginning of an integration. Consider the following diagram of the axis of the independent variable x



If the values of the y_i are computed at the points p_1 , p_2 , and p_3 using the RKB method and the truncation error test (11) calls for decreasing $|h|$ at the point p_4 , then the routine returns to the point p_0 and again computes three new points with the RKB method using the decreased value of $|h|$. If on the other hand (11a) holds at p_4 and the y_i at p_3 had been computed using the AM method, then the routine returns to the point p_3 for a new start. If the inequality in

(11c) is not satisfied at p_1 , but is satisfied at p_2 , p_3 , and p_4 , then a new start is initiated at p_4 with the increased value of $|h|$.

The user must provide a starting value for h and he may, if desired, specify a maximum value of $|h|$ beyond which the routine will not increase $|h|$ and a minimum value of $|h|$ below which it will not decrease $|h|$. Negative values of h may be supplied for backward integration.

Both the RKB method and the AM method incorporate round-off control features. This is performed in the RKB method by carrying the q 's in formula (3)-(7). In the AM method this is done by keeping the y_{in} in double precision forming the sums $y_{in} + \nabla y_{in}$ in (8) and (9) in double precision. The derivative calculations are all performed in single precision. Both procedures have shown to be very effective in controlling the growth of round-off errors.

USAGE AND CODING INFORMATION (APPENDIX B)

There are two entries to this routine. The first must be used once at the beginning to set up the routine for integration of a given set of N differential equations. The second entry may be used any number of times after the first to integrate all y_i from x to $x + h$. The first entry has the following calling sequence.

<u>Loc.</u>	<u>Instruction</u>	<u>Comment</u>
A1-2	TSX DE2F, 4	Setup entry
A1-1	PZE T, 0, V	Parameter word with addresses
A1	(Binary integer)	Option word (= 0 or 1 or 2)
A2	(Floating-point number)	\overline{E} } Truncation error testing
A3	(Floating-point number)	M } information
A4	(Floating-point number)	A
A5	(Floating-point number)	h_{\max} }
A6	(Floating-point number)	h_{\min} } Bounds on h , if any
A7	(Floating-point number)	β - Increase or decrease h factor
A7+1	Return	

The eight parameter words have the following meaning, (A1-1): V is the address of the first word of a block of $12N + 3$ cells, reserved by the user, with the arrangement

<u>Loc.</u>	<u>Contents</u>	
V	N	Fixed point binary integer, point at right
V + 1	x	Value of independent variable in floating point
V + 2	h	Value of step size in floating point
V + 3	y_1	
V + 4	y_2	
.	.	
.	.	
V + N + 2	y_N	Values of the y_i in floating point

<u>Loc.</u>	<u>Contents</u>	
V + N + 3	y ₁ '	
V + N + 4	y ₂ '	
.		
.		
V + 2N + 2	y _N '	
V + 2N + 3		Locations where the user's auxiliary subroutine must place the derivatives y _i ' .
etc.		

10 N cells of temporary storage

[Note: If the Runge-Kutta only option (see under A1 below) is used, it is only necessary to reserve a block of 4N + 3 cells].

Before executing the setup entry, the user must have already placed the appropriate numbers in cells V through V + N + 2.

The address V in the entry point of an auxiliary subroutine which the user must provide to evaluate the derivatives y_i' and store them in cells V + N + 3 through V + 2N + 2 as shown above. This auxiliary subroutine is entered by the calling sequence

<u>Loc.</u>	<u>Instruction</u>
A1-2	TSX V, 4
A1-1	Return

The setup entry uses the auxiliary subroutine to evaluate the derivatives for the initial data.

(A1): The option word may have any one of three values which designate three different modes of operation for RWINT

A1 = 0 designates the predictor-corrector variable h mode

A1 = 1 designates the fixed h Runge-Kutta only mode

A1 = 2 designates the fixed h predictor-corrector mode

For A1 = 1 or 2, the contents of A2 through A7 may be arbitrary.

(A2): This cell contains the upper bound $\bar{E} > 0$ for the truncation error testing done in the predictor-corrector variable h mode.
 $(10^{-8} \leq E \leq 10^{-3})$

(A3): This cell contains the number $M > 0$ from which the lower bound E is calculated. If $A3 = 0$, M is set equal to 100.

(A4): This cell contains the number $A > 0$ used to designate a fixed-point truncation error test as described in Appendix A. If $A4 = 0$, A is set equal to 1.

(A5): This cell may contain the upper bound $h_{\max} > 0$ for $|h|$. If $A5 = 0$, this means that there is to be no upper bound for $|h|$.

(A6): This cell may contain the lower bound $h_{\min} > 0$ for $|h|$. If $A6 = 0$, this means that there is to be no lower bound for $|h|$.

(A7): This cell may contain the factor $1 > \beta > 0$ used to increase or decrease $|h|$. If $A7 = 0$, β is set equal to 1/2.

The integration entry is quite simple and has the calling sequence

<u>Loc.</u>	<u>Instruction</u>
A1-2	TSX DE2F + 1, 4 Integration entry
A1-1	Return

Ordinarily, after execution of the integration entry, all y_i assume new values, x will have been advanced to the value $x + h$ and h will be unchanged. However, in the variable h mode, three other things can happen. (1) If the truncation error test indicates that $|h|$ should be increased, h will have been changed to $\beta^{-1}h$ unless $|\beta^{-1}h| > h_{\max}$. If the truncation error test indicates that $|h|$ should be decreased, then h will have been changed to βh unless $|\beta h| < h_{\min}$ and either (2) y_i and x will remain as they were before entry or (3) x will be changed to $x - 3h$ and the corresponding y_i values will occupy cells V + 3 through V + N + 2. Case (3) can only happen when successive decreases in $|h|$ are called for. On exit the values y_i in V + N + 3 etc. are always those which correspond to the x and y_i in V + 1 and V + 3 etc.

The integration entry must be used for each integration step. In the variable h mode, a particular integration step may involve

either AM or RKB integration but not both. In the fixed h predictor-corrector mode, the first three integration entries involve RKB integration and all subsequent ones involve AM integration.

Whenever an integration step involves AM integration, the truncation error estimate E_{n+1} is in the accumulator on exit. Zero is always placed in the accumulator if the step involved RKB integration.

The setup entry may be used again at any time to set up another problem or to change the mode of operation.

In addition to the auxiliary subroutine for derivative evaluation and the $12N + 3$ cells for data storage, the storage requirements are 693 words for RWINT plus 2 words of COMMON.

	ORG 0	INT 0001
DAUX	BCD 1DAUX	INT 0002
	HTR	INT 0003
	HTR	INT 0004
	HTR	INT 0005
INT	SXD INT-3,4	INT 0006
	SXD INT-2,2	INT 0007
	SXD INT-1,1	INT 0008
	CLA 1,4	INT 0009
	STA REV1+1	INT 0010
	STA REV1+2	INT 0011
	STA A2	INT 0012
	CLA 2,4	INT 0013
	STA A1	INT 0014
A1	CLA	INT 0015
	ARS 18	INT 0016
A2	STO	INT 0017
	ALS 2	INT 0018
	STO C	INT 0019
	ALS 1	INT 0020
	ADD C	INT 0021
	ADD C1	INT 0022
	STO C	INT 0023
	CLA 1,4	INT 0024
	SUB C	INT 0025
	STA PAR1	INT 0026
	CLA C	INT 0027
	ARS 1	INT 0028
	STO C	INT 0029
	ADD PAR1	INT 0030
	STA REV1	INT 0031
	STA REV1+3	INT 0032
	LXA C2,1	INT 0033
A4	CLA 3,4	INT 0034
	STA A5	INT 0035
A5	CLA	INT 0036
	STO PAR8+1,1	INT 0037
	TXI A6,4,-1	INT 0038
A6	TIX A4,1,1	INT 0039
	CLA PAR2	INT 0040
	LRS 18	INT 0041
	STO PAR2	INT 0042
	CLA DAUX	INT 0043
	STA AUX+2	INT 0044
	TSX REV,4	INT 0045
	TSX DE2F,4	INT 0046
PAR1	PZE 0,0,AUX	INT 0047
PAR2	PZE	INT 0048
PAR3	PZE	INT 0049
PAR4	PZE	INT 0050
PAR5	PZE	INT 0051
PAR6	PZE	INT 0052
PAR7	PZE	INT 0053

PAB8	PZE	
	TSX REV,4	INT 0054
	LXD INT-2,2	INT 0055
	LXD INT-1,1	INT 0056
	LXD INT-3,4	INT 0057
	TRA 10,4	INT 0058
INTM	SXD INT-3,4	INT 0059
	SXD INT-2,2	INT 0060
	SXD INT-1,1	INT 0061
	TSX REV,4	INT 0062
	TSX DE2F+1,4	INT 0063
	TSX REV,4	INT 0064
	LXD INT-3,4	INT 0065
	LXD INT-2,2	INT 0066
	LXD INT-1,1	INT 0067
	TRA 1,4	INT 0068
REV	LXA C,1	INT 0069
	LXD C1,2	INT 0070
REV1	CLA 0,1	INT 0071
	LDO 0,2	INT 0072
	STO 0,2	INT 0073
	STO 0,1	INT 0074
	TXI REV3,2,1	INT 0075
REV3	TIX REV1,1,1	INT 0076
	TRA 1,4	INT 0077
AUX	SXD C3,4	INT 0078
	TSX REV,4	INT 0079
	TSX 0,4	INT 0080
	TSX REV,4	INT 0081
	LXD C3,4	INT 0082
	TRA 1,4	INT 0083
C	PZE	INT 0084
C1	DEC 2	INT 0085
C2	DEC 7	INT 0086
C3		INT 0087
DE2F	REM FLOATING POINT ADAMS-MOULTON. RUNGE-KUTTA INTEGRATION	INT 0088
	TRA DE2F+0293	SETUP ENTRY
	SXD DE2F+0240,1	INT 0091
	SXD DE2F+0241,2	INT 0092
	SXD DE2F+0242,4	INT 0093
	TRA DE2F+0005	SWITCH 1
	CLA DE2F+0285	INT 0094
	CAS DE2F+0228	INT 0095
	TRA DE2F+0186	INT 0096
	TRA DE2F+0175	INT 0097
	LXA DE2F+0229,1	INT 0098
	CLA *,1	INT 0099
	STO *,1	INT 0100
	CLA *,1	INT 0101
	STO *,1	INT 0102
	CLA *,1	INT 0103
	STO *,1	INT 0104
	TIX DE2F+0010,1,1	INT 0105
		INT 0106
	Y PRIMES TO D	
	AND	
	DOUBLE PRECISION	
	YS TO TS2	
	END 3F LOOP	

LXA DE2F+0229,1	INT 0107
LDO DE2F+0223	INT 0108
FMP *,1	INT 0109
STO COMMON+000	INT 0110
CLA *,1	INT 0111
STO DE2F+0267	INT 0112
CLA *,1	INT 0113
STO DE2F+0268	INT 0114
LDO DE2F+0222	INT 0115
FMP *,1	INT 0116
FAD COMMON+000	INT 0117
STO COMMON+000	INT 0118
LDO DE2F+0221	INT 0119
FMP *,1	INT 0120
FAD COMMON+000	INT 0121
STO COMMON+000	INT 0122
LDO DE2F+0220	INT 0123
FMP *,1	INT 0124
FAD COMMON+000	INT 0125
STO COMMON+000	INT 0126
LDO *	INT 0127
FMP COMMON+000	INT 0128
TSX DE2F+0269,2	INT 0129
STO *,1	INT 0130
TIX DE2F+0018,1,1	INT 0131
CLA *	INT 0132
FAD *	INT 0133
STO *	INT 0134
LXA DE2F+0229,1	INT 0135
CLA *,1	INT 0136
STO *,1	INT 0137
TIX DE2F+0046,1,1	INT 0138
TSX 0,4	INT 0139
LXA DE2F+0229,1	INT 0140
LDO DE2F+0227	INT 0141
FMP *,1	INT 0142
STO COMMON+000	INT 0143
CLA *,1	INT 0144
STO DE2F+0267	INT 0145
CLA *,1	INT 0146
STO DE2F+0268	INT 0147
LDO DE2F+0226	INT 0148
FMP *,1	INT 0149
FAD COMMON+000	INT 0150
STO COMMON+000	INT 0151
LDO DE2F+0225	INT 0152
FMP *,1	INT 0153
FAD COMMON+000	INT 0154
STO COMMON+000	INT 0155
LDO DE2F+0224	INT 0156
FMP *,1	INT 0157
FAD COMMON+000	INT 0158
STO COMMON+000	INT 0159

D3 SUB I

PLANT Y SUB I

IN SFA

SUBROUTINE

D2 SUB I

D1 SUB I

D SUB I

LOAD H

DELTA YI UPPER P

ADD TO YI

STORE IN TS1

END OF LOOP

X+H

END OF LOOP

EVALUATE DERIVATIVES

D2 SUB I

YI FROM TS2

DP EXT.

D1 SUBI

D SUBI

Y PRIME SUBI

LDD *	LOAD H	INT 0160
FMP COMMON+000	DELTA YI UPPER C	INT 0161
TSX DE2F+0269,2	ADD TO Y I	INT 0162
STO *,1		INT 0163
STQ *,1		INT 0164
TIX DE2F+0051,1,1	END OF LOOP	INT 0165
STZ DE2F+0230		INT 0166
LXA DE2F+0229,1		INT 0167
CLA *,1		INT 0168
STO DE2F+0243		INT 0169
CLA *,1		INT 0170
STO DE2F+0244		INT 0171
TSX DE2F+0246,2		INT 0172
TIX DE2F+0078,1,1	END OF LOOP	INT 0173
TRA DE2F+0085	SWITCH 2	INT 0174
CLA DE2F+0234		INT 0175
CAS DE2F+0230		INT 0176
TRA DE2F+0090		INT 0177
TRA DE2F+0089		INT 0178
TRA DE2F+0115		INT 0179
CLA DE2F+0235		INT 0180
CAS DE2F+0230		INT 0181
TRA DE2F+0164	DECREASE H SWITCH	INT 0182
TRA DE2F+0211		INT 0183
TRA DE2F+0211		INT 0184
LXA DE2F+0229,1		INT 0185
CLA *,1		INT 0186
STO *,1	D2 TO D3	INT 0187
CLA *,1		INT 0188
STO *,1	D1 TO D2	INT 0189
CLA *,1		INT 0190
STO *,1	D TO D1	INT 0191
TIX DE2F+0096,1,1	END OF LOOP	INT 0192
CLA DE2F+0228		INT 0193
ADD DE2F+0284		INT 0194
STO DE2F+0228		INT 0195
TSX 0,4	EVALUATE DERIVATIVES	INT 0196
CLA DE2F+0230		INT 0197
FDP DE2F+0290		INT 0198
STQ COMMON+000		INT 0199
CLA COMMON+000		INT 0200
LXD DE2F+0240,1		INT 0201
LXD DE2F+0241,2		INT 0202
LXD DE2F+0242,4		INT 0203
TRA 1,4	EXIT	INT 0204
CLA *		INT 0205
SSP		INT 0206
CAS DE2F+0238	TEST H WITH HMIN	INT 0207
TRA DE2F+0121		INT 0208
TRA DE2F+0121		INT 0209
TRA DE2F+0211		INT 0210
LDD *		INT 0211
STQ DE2F+0233	STORE OLD H	INT 0212

FMP DE2F+0239	BETA TIMES H	INT 0213
STO *		INT 0214
CLA DE2F+0285		INT 0215
CAS DE2F+0228		INT 0216
HTR DE2F+0127		INT 0217
TRA DE2F+0130		INT 0218
TRA DE2F+0144		INT 0219
LDO DE2F+0289		INT 0220
FMP DE2F+0233	4H	INT 0221
CHS		INT 0222
FAD *	X-4H	INT 0223
STO *		INT 0224
LXA DE2F+0229,1		INT 0225
CLA *,1		INT 0226
STO *,1		INT 0227
CLA *,1		INT 0228
STO *,1		INT 0229
CLA *,1		INT 0230
STO *,1		INT 0231
TIX DE2F+0136,1,1	END OF LOOP	INT 0232
TRA DE2F+0161	JUMP TO SET ALPHA	INT 0233
CLA DE2F+0233		INT 0234
CHS		INT 0235
FAD *	X-H	INT 0236
STO *		INT 0237
LXA DE2F+0229,1		INT 0238
CLA *,1		INT 0239
STO *,1		INT 0240
CLA *,1		INT 0241
STO *,1		INT 0242
CLA *,1		INT 0243
STO *,1		INT 0244
TIX DE2F+0149,1,1		INT 0245
LXA DE2F+0229,1		INT 0246
LDO DE2F+0291		INT 0247
FMP *,1		INT 0248
STO *,1		INT 0249
TIX DE2F+0157,1,1	END OF LOOP	INT 0250
STZ DE2F+0228		INT 0251
STZ DE2F+0231		INT 0252
TRA DE2F+0107		INT 0253
CLA *		INT 0254
SSP		INT 0255
CAS DE2F+0237	TEST H WITH HMAX	INT 0256
TRA DE2F+0095	GO TO SHIFT DS	INT 0257
TRA DE2F+0213		INT 0258
TRA DE2F+0213		INT 0259
CLA *		INT 0260
FDP DE2F+0239	H DIVIDED BY BETA	INT 0261
STO *		INT 0262
TSX 0,4	EVALUATE DERIVATIVES	INT 0263
TRA DE2F+0156		INT 0264
LXA DE2F+0229,1		INT 0265

CLA *,1	INT 0266	
FDP DE2F+0291	INT 0267	
CLA *,1	INT 0268	
STQ COMMON+000	INT 0269	
FAD COMMON+000	INT 0270	
STO *,1	INT 0271	
STQ *,1	INT 0272	
TIX DE2F+0176,1,1	INT 0273	
TRA DE2F+0009	INT 0274	
STO *,1	INT 0275	
LDQ DE2F+0229	INT 0276	
MPY DE2F+0228	INT 0277	
STQ COMMON+000	INT 0278	
CLA COMMON+000	INT 0279	
ADD DE2F+0185	INT 0280	
STO DE2F+0194	INT 0281	
LXA DE2F+0229,1	INT 0282	
CLA *,1	INT 0283	
STO 0	STORE DERIVATIVES	INT 0284
TIX DE2F+0193,1,1	INT 0285	
CLA DE2F+0228	INT 0286	
TNZ DE2F+0204	INT 0287	
LXA DE2F+0229,1	INT 0288	
CLA *,1	INT 0289	
STO *,1	INT 0290	
CLA *,1	INT 0291	
STO *,1	INT 0292	
TIX DE2F+0199,1,1	INT 0293	
CLA DE2F+0228	INT 0294	
ADD DE2F+0284	ALPHA PLUS ONE	INT 0295
STO DE2F+0228	RUNGA-KUTTA ENTRY	INT 0296
TSX DE2F+0454,4	INT 0297	
STZ COMMON+000	INT 0298	
CLA COMMON+000	ZERO TO ACC	INT 0299
TRA DE2F+0111	INT 0300	
STZ DE2F+0231	INT 0301	
TRA DE2F+0095	INT 0302	
CLA DE2F+0231	INT 0303	
ADD DE2F+0284	R+1	INT 0304
STO DE2F+0231	INT 0305	
CAS DE2F+0292	INT 0306	
HTR DE2F+0217	INT 0307	
TRA DE2F+0170	INT 0308	
TRA DE2F+0095	INT 0309	
DEC 2.291666667	INT 0310	
DEC -2.458333333	INT 0311	
DEC 1.541666667	INT 0312	
DEC -3.75E-1	INT 0313	
DEC 3.75E-1	INT 0314	
DEC 7.916666667E-1	INT 0315	
DEC -2.083333333E-1	INT 0316	
DEC 4.166666667E-2	INT 0317	
BSS 4	INT 0318	

BSS 8	INT 0319
BSS 3	INT 0320
HTR	INT 0321
HTR	INT 0322
HTR	INT 0323
CLA DE2F+0243	INT 0324
SSP	INT 0325
CAS DE2F+0236	INT 0326
TRA DE2F+0254	INT 0327
TRA DE2F+0254	INT 0328
CLA DE2F+0236	INT 0329
STO DE2F+0245	INT 0330
TRA DE2F+0255	INT 0331
STO DE2F+0245	INT 0332
CLA DE2F+0244	INT 0333
FSB DE2F+0243	INT 0334
SSP	INT 0335
FDP DE2F+0245	INT 0336
STQ COMMON+000	INT 0337
CLA DE2F+0230	INT 0338
CAS COMMON+000	INT 0339
TRA 1,2	INT 0340
TRA 1,2	INT 0341
CLA COMMON+000	INT 0342
STO DE2F+0230	INT 0343
TRA 1,2	INT 0344
HTR	INT 0345
HTR	INT 0346
UFA DE2F+0267	INT 0347
STO DE2F+0267	INT 0348
STQ COMMON+000	INT 0349
CLA COMMON+000	INT 0350
UFA DE2F+0268	INT 0351
FAD DE2F+0267	INT 0352
TRA 1,2	INT 0353
TRA DE2F+0005	INT 0354
TRA DE2F+0207	INT 0355
TRA DE2F+0085	INT 0356
TRA DE2F+0095	INT 0357
TRA DE2F+0115	INT 0358
TRA DE2F+0121	INT 0359
TRA DE2F+0164	INT 0360
TRA DE2F+0213	INT 0361
DEC 1	INT 0362
DEC 3	INT 0363
DEC 1.	INT 0364
DEC 5E-1	INT 0365
DEC 100.	INT 0366
DEC 4.	INT 0367
DEC 14.	INT 0368
DEC -3.	INT 0369
DEC 3	INT 0370
SXD DE2F+0241,2	INT 0371
HI UPPER C	
YI UPPER P	
DIVISOR	
ENTRY	
TEST FOR DIVISOR	
A1	
A2	
ENTRY	
SPECIAL	
FLOATING	
ADDITION	
SUBROUTINE	
SWITCH 1, A LEG	
SWITCH 1, B LEG	
SWITCH 2, A LEG	
SWITCH 2, B LEG	
DECREASE H SWITCH, TEST LEG	
DECREASE H SWITCH, NO TEST LEG	
INCREASE H SWITCH, TEST LEG	
INCREASE H SWITCH, NO TEST LEG	
ONE HALF	
SPECIAL TEST NO. FOR R	

SXD DE2F+0242,4	INT 0372
CLA DE2F+0276	INT 0373
STO DE2F+0004	INT 0374
CLA DE2F+0278	INT 0375
STO DE2F+0084	INT 0376
CLA DE2F+0280	INT 0377
STO DE2F+0089	INT 0378
CLA DE2F+0282	INT 0379
STO DE2F+0092	INT 0380
CLA 2,4	INT 0381
PAX 0,2	INT 0382
TRA DE2F+0308,2	INT 0383
TRA DE2F+0346	INT 0384
TRA DE2F+0343	INT 0385
LDQ 3,4	INT 0386
FMP DE2F+0290	INT 0387
STO DE2F+0234	INT 0388
CLA 8,4	INT 0389
TNZ DE2F+0314	INT 0390
CLA DE2F+0287	INT 0391
STO DE2F+0239	INT 0392
CLA 4,4	INT 0393
TNZ DE2F+0318	INT 0394
CLA DE2F+0288	INT 0395
STO DE2F+0235	INT 0396
CLA DE2F+0234	INT 0397
FDP DE2F+0235	INT 0398
STQ DE2F+0235	INT 0399
CLA 5,4	INT 0400
TNZ DE2F+0325	INT 0401
CLA DE2F+0286	INT 0402
STO DE2F+0236	INT 0403
CLA 6,4	INT 0404
TNZ DE2F+0331	INT 0405
CLA DE2F+0283	INT 0406
STO DE2F+0092	INT 0407
TRA DE2F+0335	INT 0408
STO DE2F+0237	INT 0409
LDQ DE2F+0239	INT 0410
FMP DE2F+0237	INT 0411
STO DE2F+0237	INT 0412
CLA 7,4	INT 0413
TNZ DE2F+0340	INT 0414
CLA DE2F+0281	INT 0415
STO DE2F+0089	INT 0416
TRA DE2F+0348	INT 0417
FDP DE2F+0239	INT 0418
STO DE2F+0238	INT 0419
TRA DE2F+0348	INT 0420
CLA DE2F+0277	INT 0421
STO DE2F+0004	INT 0422
TRA DE2F+0348	INT 0423
CLA DE2F+0279	INT 0424
SET SWITCHES TO NORMAL POSITIONS	
SELECT OPTION	
OPTION 2	
OPTION 1	
OPTION 0	
14E (UPPER)	
TEST BETA	
STORE BETA	
STORE M	
STORE 14E (LOWER)	
TEST A	
STORE A	
TEST H MAX	
STORE BETA (H MAX)	
TEST H MIN	

STO DE2F+0084	INT 0425
CLA 1,4	INT 0426
STO DE2F+0232	INT 0427
STO DE2F+0352	INT 0428
TSX DE2F+0453,4	INT 0429
PZE	INT 0430
STZ DE2F+0228	INT 0431
STZ DE2F+0231	INT 0432
CLA DE2F+0232	INT 0433
STA DE2F+0357	INT 0434
CLA 0	INT 0435
STO DE2F+0229	INT 0436
CLA DE2F+0232	INT 0437
ARS 18	INT 0438
STA DE2F+0049	INT 0439
STA DE2F+0106	INT 0440
STA DE2F+0173	INT 0441
CLA DE2F+0232	INT 0442
ADD DE2F+0284	INT 0443
STA DE2F+0042	INT 0444
STA DE2F+0044	INT 0445
STA DE2F+0133	INT 0446
STA DE2F+0134	INT 0447
STA DE2F+0146	INT 0448
STA DE2F+0147	INT 0449
ADD DE2F+0284	INT 0450
STA DE2F+0037	INT 0451
STA DE2F+0043	INT 0452
STA DE2F+0070	INT 0453
STA DE2F+0115	INT 0454
STA DE2F+0121	INT 0455
STA DE2F+0124	INT 0456
STA DE2F+0164	INT 0457
STA DE2F+0170	INT 0458
STA DE2F+0172	INT 0459
ADD DE2F+0284	INT 0460
ADD DE2F+0229	INT 0461
STA DE2F+0010	INT 0462
STA DE2F+0021	INT 0463
STA DE2F+0047	INT 0464
STA DE2F+0073	INT 0465
STA DE2F+0078	INT 0466
STA DE2F+0139	INT 0467
STA DE2F+0152	INT 0468
STA DE2F+0178	INT 0469
STA DE2F+0181	INT 0470
STA DE2F+0199	INT 0471
ADD DE2F+0229	INT 0472
STA DE2F+0012	INT 0473
STA DE2F+0067	INT 0474
STA DE2F+0137	INT 0475
STA DE2F+0150	INT 0476
STA DE2F+0193	INT 0477

SETUP RK SUBROUTINE
PARAMETER WORD
SET ALPHA TO ZERO
R 0

STORE N

SETUP
DERIVATIVE
EVALUATIONS

T=1

T=2

T=3 EQUALS D
D=N

D+2N

RM 61TMP-32

ADD DE2F+0229	D+3N	INT 0478
STA DE2F+0014		INT 0479
STA DE2F+0023		INT 0480
STA DE2F+0056		INT 0481
STA DE2F+0074		INT 0482
STA DE2F+0141		INT 0483
STA DE2F+0154		INT 0484
STA DE2F+0158		INT 0485
STA DE2F+0159		INT 0486
STA DE2F+0176		INT 0487
STA DE2F+0182		INT 0488
STA DE2F+0201		INT 0489
ADD DE2F+0229	D+4N	INT 0490
STA DE2F+0040		INT 0491
STA DE2F+0046		INT 0492
STA DE2F+0080		INT 0493
ADD DE2F+0229	D+5N	INT 0494
STA DE2F+0011		INT 0495
STA DE2F+0054		INT 0496
STA DE2F+0151		INT 0497
ADD DE2F+0229	D+6N	INT 0498
STA DE2F+0015		INT 0499
STA DE2F+0153		INT 0500
ADD DE2F+0229	D+7N	INT 0501
STA DE2F+0138		INT 0502
STA DE2F+0200		INT 0503
ADD DE2F+0229	D+8N	INT 0504
STA DE2F+0140		INT 0505
STA DE2F+0202		INT 0506
ADD DE2F+0229	D+9N	INT 0507
STA DE2F+0019		INT 0508
STA DE2F+0097		INT 0509
STA DE2F+0136		INT 0510
STA DE2F+0185		INT 0511
ADD DE2F+0229	D+10N	INT 0512
STA DE2F+0026		INT 0513
STA DE2F+0052		INT 0514
STA DE2F+0096		INT 0515
STA DE2F+0099		INT 0516
ADD DE2F+0229	D+11N	INT 0517
STA DE2F+0030		INT 0518
STA DE2F+0059		INT 0519
STA DE2F+0098		INT 0520
STA DE2F+0101		INT 0521
ADD DE2F+0229	D+12N	INT 0522
STA DE2F+0013		INT 0523
STA DE2F+0034		INT 0524
STA DE2F+0063		INT 0525
STA DE2F+0100		INT 0526
STA DE2F+0149		INT 0527
LXD DE2F+0241,2		INT 0528
LXD DE2F+0242,4		INT 0529
TRA 9,4	EXIT	INT 0530

TRA DE2F+0562	TO SETUP REGION	INT 0531
SXD DE2F+0553,1	SAVE INDEX REGISTERS	INT 0532
SXD DE2F+0554,2	FROM	INT 0533
SXD DE2F+0555,4	MAIN PROGRAM	INT 0534
CLA		INT 0535
FDP DE2F+0520	CALCULATE	INT 0536
STQ DE2F+0525	H DIVIDED BY 2	INT 0537
STQ DE2F+0541		INT 0538
LXA DE2F+0551,2	SET PARAMETER INDEX	INT 0539
LXA DE2F+0556,1	SET N INDEX.	INT 0540
LDQ ,1		INT 0541
FMP DE2F+0551,2	CALCULATE NEW	INT 0542
STO ,1	VALUE OF P	INT 0543
LDQ		INT 0544
FMP ,1	CALCULATE NEW	INT 0545
FAD ,1	VALUE OF R	INT 0546
STO ,1		INT 0547
FDP DE2F+0552,2	CALCULATE NEW	INT 0548
STQ COMMON+000	VALUE OF R	INT 0549
LDQ ,1		INT 0550
FMP DE2F+0553,2	CALCULATE NEW	INT 0551
FAD COMMON+000	VALUE OF R	INT 0552
STO COMMON+000		INT 0553
TZE DE2F+0492	TEST VALUES	INT 0554
ARS 27		INT 0555
STO COMMON+001	TO	INT 0556
CLA ,1	DETERMINE	INT 0557
TZE DE2F+0492	IF SHIFTING	INT 0558
SSP		INT 0559
ARS 27	IS NECESSARY	INT 0560
SBM COMMON+001		INT 0561
TMI DE2F+0492	CALCULATE NEW	INT 0562
TZE DE2F+0492	VALUE OF Z	INT 0563
STA DE2F+0489		INT 0564
STA DE2F+0490	CALCULATE	INT 0565
CLA COMMON+000		INT 0566
ARS	NEW VALUE	INT 0567
ALS		INT 0568
STO COMMON+000	OF Q	INT 0569
CLA ,1		INT 0570
FAD COMMON+000	CALCULATE NEW	INT 0571
STO ,1	VALUE OF Z	INT 0572
TRA DE2F+0558,2		INT 0573
LDQ ,1	CALCULATE	INT 0574
FMP DE2F+0556,2		INT 0575
STO ,1	NEW VALUE	INT 0576
LDQ ,1		INT 0577
FMP DE2F+0555,2	CALCULATE	INT 0578
FAD ,1		INT 0579
STO ,1	NEW VALUE	INT 0580
LDQ COMMON+000		INT 0581
FMP DE2F+0554,2	CALCULATE	INT 0582
FAD ,1		INT 0583

STO ,1		INT 0584
TIX DE2F+0463,1,1	TEST N	INT 0585
CLA		INT 0586
FAD DE2F+0557,2	INCREASE X	INT 0587
STO		INT 0588
SXD DE2F+0557,2		INT 0589
TSX ,4	FIND DERIVITIVES	INT 0590
LXD DE2F+0557,2		INT 0591
TIX DE2F+0462,2,8	TEST PASS NO.	INT 0592
LXD DE2F+0553,1	RESTORE	INT 0593
LXD DE2F+0554,2	INDEX	INT 0594
LXD DE2F+0555,4	REGISTERS	INT 0595
TRA 1,4		INT 0596
DEC 0		INT 0597
DEC 2.		INT 0598
DEC -1.		INT 0599
DEC 3.		INT 0600
DEC -.5		INT 0601
DEC 1.		INT 0602
DEC		INT 0603
TRA DE2F+0496		INT 0604
DEC 0		INT 0605
DEC 2.		INT 0606
DEC -.5		INT 0607
DEC -1.		INT 0608
DEC .5		INT 0609
DEC -3.		INT 0610
DEC 0		INT 0611
TRA DE2F+0558		INT 0612
DEC -.5		INT 0613
DEC 1.		INT 0614
DEC 0		INT 0615
DEC -1.		INT 0616
DEC 0		INT 0617
DEC 1.		INT 0618
DEC		INT 0619
TRA DE2F+0496		INT 0620
DEC 2.		INT 0621
DEC 6.		INT 0622
DEC 1.		INT 0623
DEC 3.		INT 0624
DEC -.5		INT 0625
DEC -3.		INT 0626
DEC 0		INT 0627
TRA DE2F+0496		INT 0628
PZE 32,,		INT 0629
PZE 1,,		INT 0630
BSS 5		INT 0631
CLA ,1		INT 0632
FDP DE2F+0556,2		INT 0633
STQ ,1		INT 0634
TRA DE2F+0499		INT 0635
SXD DE2F+0553,1	SAVE INDEX REGISTERS	INT 0636

SXD DE2F+0555+4	FROM MAIN PROGRAM	INT 0637
CLA 1,4		INT 0638
STA DE2F+0553		INT 0639
STA DE2F+0570		INT 0640
ARS 18		INT 0641
STA DE2F+0512	SET ADDRESS OF	INT 0642
STA DE2F+0603	DERIVITIVE ROUTINE	INT 0643
CLA	STORE VALUE	INT 0644
STO DE2F+0556	OF N	INT 0645
CLA DE2F+0553		INT 0646
ADD DE2F+0552	STORE ADDRESS	INT 0647
STA DE2F+0508	OF X	INT 0648
STA DE2F+0510		INT 0649
ADD DE2F+0552	STORE ADDRESS	INT 0650
STA DE2F+0466	OF H	INT 0651
STA DE2F+0457		INT 0652
ADD DE2F+0552	STORE ADDRESS	INT 0653
ADD DE2F+0556	OF Y	INT 0654
STA DE2F+0479		INT 0655
STA DE2F+0492	STORE ADDRESS	INT 0656
STA DE2F+0494	OF DERIVITIVE	INT 0657
ADD DE2F+0556		INT 0658
STA DE2F+0467	STORE	INT 0659
ADD DE2F+0556	ADDRESS	INT 0660
STA DE2F+0605		INT 0661
STA DE2F+0560	OF	INT 0662
STA DE2F+0558		INT 0663
STA DE2F+0472	Q	INT 0664
STA DE2F+0496		INT 0665
STA DE2F+0498	STORE ADDRESS	INT 0666
STA DE2F+0501	OF P	INT 0667
STA DE2F+0502		INT 0668
STA DE2F+0503	FIND INITIAL DERIVITIVES	INT 0669
STA DE2F+0506		INT 0670
ADD DE2F+0556	SET ORGINAL Q	INT 0671
STA DE2F+0463	TO ZERO	INT 0672
STA DE2F+0465	RESTORE INDEX	INT 0673
STA DE2F+0468	REGISTERS	INT 0674
STA DE2F+0469		INT 0675
STA DE2F+0499		INT 0676
TSX ,4		INT 0677
LXA DE2F+0556+1		INT 0678
STZ ,1		INT 0679
TIX DE2F+0605+1,1		INT 0680
LXD DE2F+0553+1		INT 0681
LXD DE2F+0555+4		INT 0682
TRA 2,4		INT 0683
COMMON BSS 2		INT 0684
R END		INT 0685

H. SUBROUTINE RINDEX

This subroutine permits the calculation of the refractive index, its spatial derivatives, and the absorption coefficient, all as functions of the local atmosphere and its state of ionization. This subroutine also permits the output of "debugging data" which is called the R vector whose components are defined in Table 3. See subroutine OUTPUT for additional information.

$R(1) = X$
 $R(2) = PXR = \frac{\partial X}{\partial r}$
 $R(3) = PXTHET = \frac{\partial X}{\partial \theta}$
 $R(4) = PXPHI = \frac{\partial X}{\partial \phi}$
 $R(5) = Y$
 $R(6) = YR = Y_r$
 $R(7) = YTHTETA = Y_\theta$
 $R(8) = YPHI = Y_\phi$
 $R(9) = PYR = \frac{\partial Y}{\partial r}$
 $R(10) = PYTHET = \frac{\partial Y}{\partial \theta}$
 $R(11) = PYPHI = \frac{\partial Y}{\partial \phi}$
 $R(12) = Z$
 $R(13) = PZR = \frac{\partial Z}{\partial r}$
 $R(14) = PZTHET = \frac{\partial Z}{\partial \theta}$
 $R(15) = PZPHI = \frac{\partial Z}{\partial \phi}$
 $R(16) = COSPSI = \cos \psi$
 $R(17) = SINPSI = \sin \psi$
 $R(18) = YSI = Y \sin \psi$
 $R(19) = YCI = Y \cos \psi$
 $R(20) = TE1 = (1 - X)^2 - Z^2$

Table 3. Nomenclature Describing the R Vector.
(Page 1 of 4)

$$R(21) = TE2 = (YSI)^4 + 4TE1(YCI)^2$$

$$R(22) = TE3 = 8(YCI)^2 Z(1 - X)$$

$$R(23) = R2S = R_S^2$$

$$R(24) = R1S = R_S$$

$$R(25) = THET2S = 2\theta_S$$

$$R(26) = THET1S = \theta_S$$

$$R(27) = S1 = S_1$$

$$R(28) = S2 = S_2$$

$$R(29) = D1 = d_1$$

$$R(30) = D2 = d_2$$

$$R(31) = TE4 = d_1^2 + d_2^2$$

$$R(32) = TE5 = 2X[Zd_1 + (1 - X)d_2]/TE4$$

$$R(33) = TE6 = 1 - [2X(1 - X)d_1 - Zd_2]/TE4$$

$$R(34) = R2M = R_M^2$$

$$R(35) = R1M = R_M$$

$$R(36) = THET2M = 2\theta_M$$

$$R(37) = THET1M = \theta_M$$

$$R(38) = AM1 = M_1$$

$$R(39) = AM2 = M_2$$

$$R(40) = TE7 = M_1d_1 - M_2d_2$$

Table 3. Nomenclature Describing the R Vector.
(Page 2 of 4)

$$R(41) = TE8 = M_1 d_2 + M_2 d_1$$

$$R(42) = AO = a_o$$

$$R(43) = BO = b_o$$

$$R(44) = TE9 = S_1^2 + S_2^2$$

$$R(45) = A4 = a_4$$

$$R(46) = B4 = b_4$$

$$R(47) = A5 = a_5$$

$$R(48) = B5 = b_5$$

$$R(49) = PNPX = \partial \mu / \partial X$$

$$R(50) = TE10 = (\sin \psi)^2 (YSI)^2 + 2TE1 \cos^2 \psi$$

$$R(51) = TE11 = 4Z(1 - X)\cos^2 \psi$$

$$R(52) = A6 = a_6$$

$$R(53) = B6 = b_6$$

$$R(54) = PNPy = \partial \mu / \partial Y$$

$$R(55) = A7 = a_7$$

$$R(56) = B7 = b_7$$

$$R(57) = PNPZ = \partial \mu / \partial Z$$

$$R(58) = A8 = a_8$$

$$R(59) = B8 = b_8$$

$$R(60) = TE13 = 1/(2 \cos^2 \theta + \frac{1}{2} \sin^2 \theta)$$

Table 3. Nomenclature Describing the R Vector.
(Page 3 of 4)

$$R(61) = A1 = a_1$$

$$R(62) = B1 = b_1$$

$$R(63) = A2 = a_2$$

$$R(64) = B2 = b_2$$

$$R(65) = TE12 = (w_1)^2 \text{ YSI}$$

$$R(66) = PPSIPT = \partial \Psi / \partial \theta$$

$$R(67) = PPSIPR = \partial \Psi / \partial r$$

$$R(68) = PPSIPP = \partial \Psi / \partial \phi$$

$$R(69) = TE14 = \sqrt{\sigma_r^2 + \sigma_\theta^2 + \sigma_\phi^2}$$

Table 3. Nomenclature Describing the R Vector.
(Page 4 of 4)

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C-----RINDEX WITH MAGNETIC FIELD DECEMBER 22, 1960 - IBM-7090
1   SUBROUTINE RINDEX
COMMON RECORD,V,B,N
DIMENSION RECORD(120),VC(112),WC(500),RC(75)
PI = 3.1415927
CALL ELECTX CX,PXR,PZTHET,EXPHI
      PXR = X
      PZTHET = PXR
      PZPHI = EXPHI
      SIGN = JCC67
      CALL KRASH CH,VR,VTHETA,VPHI,PF,PZTHET,ZPHI
      R03 = V
      R02 = VR
      R01 = VTHETA
      R00 = VPHI
      P03 = PVY
      P02 = PYTHET
      P01 = PZPHI
      R010 = PZTHET
      R011 = PZPHI
      CALL COLFRZC Z,PZR,PZTHET,PZPHI;
      R012 = Z
      R013 = PZR
      R014 = PZTHET
      R015 = PZPHI
      WC650 = X
      WC650 = Y
      YC640 = Z
      C0SPSI = VC(67)*VP + VC(68)*VTHETA + VC(69)*VPHI/CP*WC150
      RC150 = C0SPSI
      SINPSI = S0RT( C 1.0 - C0SPSI*C0SPSI )
      RC170 = SINPSI
      VSI = VASINPSI
      RC180 = VSI
      VC1 = V*C0SPSI
      RC190 = YCI
      IF(CZ-1.0E-180 9,10,10
      T = 0.0
      G 70
      10 T1 = Z#Z
      11 TE1 = CC1.0- X#*C1.0- : - 2
      RC200 = TEL
      TEE = VSI*VSI
      TEA = VCL*VCL
      12 TE2 = TEE*TEE + 4.0*TER*TE1
      RC210 = TE2
      TEB = 2*C1.0-X#
      13 TE3 = 8.0*TER*TER
      RC220 = TE3
      14 R35 = TE2*TE2 + TE3*TE3
      P25 = S0RT(P45)
      RC23 = P25
      15 R15 = S0RT CP25
      16 THET25 = DATA(THET2,-TE3)
      IF(CP1 - THET25>200,209,17
      200 R15 = -R15
      17 THET15 = 0.9*THET25
      RC240 = R15
      RC250 = THET25

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R(249) = THET15
 S1 = R15 + COS(THET15)*SIGN
 18 R(277) = S1
 19 S2 = R15 + SIN(THET15)*SIGN
 R(295) = S2
 20 D1 = 2.0*X1-0.0*Y2 - TEE + S1
 R(297) = D1
 21 D2 = S2 + C1-0.0*X1-0.0*Y2
 P(309) = D2
 22 TE0 = C1+C1 + DC*DC
 PC(311) = TE0*
 23 TE5 = 2.0*X1*DC*DC+C1-0.0*D20 /TE2*
 P(322) = TE5
 TE0 = C1-0.0*X0+D1
 TE5 = C1-0.0*X0+CTEC-2+DC1 - TE4
 P(333) = TE5
 P(341) = TE6+TE5 + TE5+TE5
 P(349) = 3QTF(CR40)
 P(351) = P2H
 P(352) = 3QTF(CP2M)
 THET14 = S1*ATAN(CTE5/TE5)
 IF(0) = -THET2M*2C1,2D1
 P14 = -R1M
 THET14 = 0.5 * THET2M
 P(353) = S1M
 P(354) = THET1M
 G14 = P1W + CCEFC(THET14)
 P(356) = S1M
 P(358) = S1W *SINF(THET14)
 P(360) = S1M
 P(361) = R11
 M12 = R1M
 TE5 = R11+C1 - RMC+DC1
 P(362) = TE7
 TE5 = R11+DC1 + RMC+DC1
 P(363) = TE2
 TED = TE7+TE7
 TE5 = TE3+TE6
 25 = TE7/CTEC + TEF
 R12D = MO
 ED = TE3*MO+TE7
 PC+3D = BG
 27 TE5 = S151 + S2*SD
 P(364) = TE9
 28 = 2.0*TE6
 TE5 = C1.0-X2*S1 - Z*52
 R1 = 1.0 + Q5*TEG/TE4
 R(450) = R4
 TEH = C2*ST1 + C1.0-X2*S2/TE9
 B4 = Z + TEH*G5
 P(450) = B4
 R5 = 2.0*X*Z*D1 + C1.0-X2*D2
 R(451) = R45
 R5 = 2.0*X-1.0 - R4*G5 + B4*E5
 TEJ = 2.0*X-1.0 - R4*G5 + B4*E5
 TEK = 2 + B4*G5 + R4*B5

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42 PNP6 = A0*TE1 + B0*TEK
    R(9) = PNPX
    Q5 = SINPS1*2
    Q6 = (COSPS1**2)*2.0
    TE10 = TEE*Q5 + Q6*TE1
    R(50) = TE10
    TE2 = 2.0*TEB
    TE11 = TEE2*Q6
    RC51 = TE11
    R5 = CTE10*S1 - TE11*S2)/TE9
    RC52 = R5
    B6 = CTE10*S2 + TE11*S1)/TE9
    RC32 = B6
    Q1 = B0*A5 + B5*A0
    Q2 = A0*A5 - B0*A5
    Q3 = A6 - SINPS1*SINPS1
    Q4 = Q2*Q3
    Q7 PNPY = V*Q4 - Q1*B6*V
    R(54) = PNPY
    Q5 = 2.0*TEA
    Q8 A7 = CC2.0-X) + Q3*TEG/TE9
    RC55 = A7
    Q9 B7 = 2.0*Q2 + TEH*TE9
    RC56 = B7
    Q1 = X - Q5*Q7 + 25*Q7
    Q2 = 1.0 - X*X
    Q4 = S1*Q3 - S2*Z*C1.0-X)
    R3 = 2.0*Q1.0 + TEA*Q3/TE9
    RC58 = R5
    R6 = S2*Q3 + 2*Q1*(1.0*Y)
    R9 = 2*(2.0+X)+ Q3*Q6/TE9
    RC592 = R8
    Q1 = 2.0*X*X - Q5*Q8 + B5*B8
    Q1 = 2.0*B8*X*X2 - Q4*B5*B8) + 2*(Q5*B3*B2
    Q2 = X*Z + A5*Q8 + B5*Q8
    Q3 = Y*Q12.5 - Q80*Q1 + B0*Q22/2*(Q32
    Q1 = COSF(CU5))
    Q2 = SINFCU522
    Q3 = 2.0*Q1*Q1 + 0.5*Q2*Q2
    TE13 = 1.0*Q3
    R(60) = TE13
    Q5 = IF(SINECY512=1,DE=162102,100,60)
    Q6 AI = 2.0* TE1 - TEE
    RC642 = A1
    R61 B1 = 4.0* TE8
    R(62) = B1
    Q2 = C1.0 + Q1*S1 - B1*S2/TE9
    RC632 = B2
    Q3 B2 = C51*B1 + A1*S2)/TE9
    R642 = B2
    TE9 = B2*Q5
    T89 = B2*Q5
    TAC = B2*Q5
    T80 = B2*Q5
    TAE = B0*CTAE-TAE
  
```

RM 61 TMP-32

STORAGE LOCATIONS FOR VARIABLES APPEARING IN COMMON STATEMENTS

DEC OCT
R 32188 73674
REC 32531 74641

INTERNAL FORMULA NUMBERS WITH CORRESPONDING INTERNAL FORMULA NUMBER AND TOTAL LOCATIONS

EFN	IFN	LOC	EFN	IFN	LOC	EFN	IFN	LOC	EFN	IFN	LOC
128	173000	16	141	CC011	14	142	CC011	14	143	CC011	14
129	173020	11	143	CC015	16	150	CC025	18	151	CC025	18
130	173040	16	152	CC026	18	153	CC026	18	154	CC032	18
131	173060	16	155	CC021	18	156	CC021	18	157	CC043	18
132	173080	24	158	CC0415	24	159	CC0415	24	160	CC0434	24
133	173100	261	161	CC042	32	162	CC0517	32	163	CC0521	32
134	173120	157	164	CC0567	42	165	CC0723	42	166	CC0614	42
135	173140	152	167	CC0137	52	168	CC0175	52	169	CC0264	52
136	173160	151	170	CC0150	61	171	CC0340	61	172	CC1340	61
137	173180	66	173	CC0457	71	174	CC0747	71	175	CC1474	71
138	173200	71	176	CC0134	102	177	CC01621	102	178	CC1542	102
139	173220	105	179	CC01556	105	180	CC01523	105	181	CC1523	105
140	173240	131	181	CC01737	131	182	CC01714	131	183	CC1714	131
141	173260	130	184	CC01737	130	185	CC01740	130	186	CC1740	130

STORAGE NOT USED BY PROGRAM

DEC OCT
R 32187 73672

LOCATIONS OF NAMES IN TRANSFER VECTOR

REC	OCT	DEC	OCT	REC	OCT	DEC	OCT	REC	OCT	DEC	OCT
ELECT	1 00000	008	1 00000	COLFRZ	2 00002	MAG	1 00201	WTRAN	2 00000	B7	11110
QIN	1 00007	014	1 00006	SQRT	2 00003	CPFLD	1 00011	WTAB	3 00000	B7	11110

STORAGE LOCATIONS FOR VARIABLES APPEARING IN DIMENSION AND EQUIVALENCE SENTENCES

DEC OCT
R 1193 02254

STORAGE LOCATIONS FOR VARIABLES NOT APPEARING IN DIMENSION, EQUIVALENCE OR COMMON SENTENCES

DEC	OCT	DEC	OCT	DEC	OCT	DEC	OCT	DEC	OCT	DEC	OCT
D2	1121 02141	D1	1120 02140	COSPI	1115 02137	B8	1118 02136	B7	11110	B7	11110
B6	1116 02134	B5	1115 02143	B4	1114 02132	B2	1113 02131	B1	1112	B1	1112
BG	1111 02127	AM2	1110 02126	AM1	1109 02125	A9	1108 02124	A7	1107	A7	1107
A6	1106 02121	AM	1105 02121	A4	1104 02120	A2	1103 02119	A1	1102	A1	1102
P0	1102 02110	P1	1102 02114	PNPX	1095 02115	FNPY	1098 02112	FNPZ	1097	FNPZ	1097
PPSIPP	1096 02110	PPSPT	1098 02109	PPSPT	1094 02108	PNPFI	1095 02105	PNPFI	1092	PNPFI	1092
PXTHET	1091 02103	PYPHI	1090 02102	PYR	1089 02101	ENTHET	1088 02100	ENTHET	1087	ENTHET	1087
PZRA	1083 02076	FZTHE	1085 02073	FZTHE	1084 02074	GZ	1083 02073	GZ	1082	GZ	1082
G4	1081 02071	AM3	1081 02070	AM3	1080 02069	F1M	1079 02068	F1M	1078	F1M	1078
R2P	1076 02064	P2E	1077 02063	P2E	1076 02062	F4S	1075 02061	F4S	1074	F4S	1074

5151	107.	UC556	SINPSI	1065	02055	T1	1068	02054	TAA	1067	02053
TE10	1065	02051	TA0	1064	02053	TAE	1063	02047	TAF	1062	02046
TE15	1060	02044	TE11	1059	02043	TE12	1058	02042	TE13	1057	02041
TE1	1055	02037	TE2	1054	02036	TE3	1053	02035	TE4	1052	02034
TE8	1050	02032	TE7	1049	02031	TE8	1048	02030	TE9	1047	02027
TE5	1045	02025	TEB	1044	02024	TEC	1043	02023	TEC	1042	02022
TEF	1041	02029	TEG	1039	02017	TEH	1038	02016	TEJ	1037	02015
-3717	1036	02013	THET1S	1034	02012	THET2M	1033	02011	THET2S	1032	02010
1	1029	02006	X2	1029	02005	X3	1028	02004	Y1	1027	02003
-1	1025	02001	YR	1024	02000	Y	1023	01777	YSI	1022	01776
-	1029	01774									

***** LOOPS FOR SYMBOLS NOT APPEARING IN SOURCE PROGRAM *****

DEC	OCT	DEC	OCT	DEC	OCT	DEC	OCT
1011	07770	2)	996	01744	3)	999	01747

***** PRINTS TO SUBROUTINES NOT OUTPUT FROM LIBRARY.
***** FORT QATAN COS SIN QM CSTM CFIL2

00000 ELECTR	BCD	1ELECIX		238	B55	COLFRZ, 4		00160 4081	TZE 43B
00001 MAGY	BCD	1MAGY		00070 26A	TSX	COLFRZ, 4		00161	TPL 43A
00002 COLFRZ	BCD	1COLFRZ		00071	TSX	2		00162 41A	CLA 32+3
00003 SORT	BCD	1SORT		00072	TSX	PZR		00163	STO T1
00004 QATAN	BCD	1QATAN		00073	TSX	PZTHET		00164 42A	TRG 44B
00005 COS	BCD	1COS		00074	TSX	PZPHI		00165 43A	LDQ 2
00006 SIN	BCD	1SIN		00075 25A	CLA	2		00166	FMP Z
00007 ON	BCD	1ON		00076	STO	R-11		00167	STO T1
00010 CSTM2	BCD	1CSTM2		00077 24A	CLA	PZR		00170 44A	CLA 32+1
00011 CP1L2	BCD	1CP1L2		00100	STO	R-12		00171	F58 X
00012 S	MTR			00101 27A	CLA	PZTHET		00172	STO 12+1
00013 MTR				00102	STO	R-13		00173	LDQ 13+1
00014 MTR				00103 28A	CLA	PZPHI		00174	FMP 12+1
00015 BCD 1RINDEX				00104	STO	R-14		00175	F5B T1
00016 SVD	S11			00105 29A	CLA	X		00176	STO TE1
00017 SVD	S11	.2		00106	STO	Y-65		00177 45A	CLA TE1
00020 SVD	SVD	.5+2.4		00107 30A	CLA	Y		00200	STO R-19
00021 6A	CLA	33		00110	STO	Y-64		00201 46A	LDQ VSI
00022 STD P1				00111 31A	CLA	2		00202	FMP YSI
7A	B55	1SX_ELECTR, 4		00112	STO	Y-63		00203	STO TEE
9-22	8	1SX_ELECTR, 4		00113 32A	LDQ	Y		00204 47A	LDQ YCI
00024 TS-X				00114	FMP	Y		00205	FMP YCI
00025 TS-X				00115	STO	J2+1		00206	STO TEE
00026 TSX_PXTHET				00116	Q	V-8		00207 48A	LDQ TE1
00027 TSX_PXPHI				00117	FMP	Y-HI		00210	FMP 32+4
00030 %A	CLA	X		00120	STO	12+2		00211	XCA
00031 STD R-1				00121	LDO	Y-7		00212	FMP -ER
00032 10A	CLA	PXR		00122	FMP	YTHETA		00213	STO 13+1
00033 STD R-1				00123	STO	J2+3		00214	LDQ TEE
00034 11A	CLA	PXTHET		00124	LDO	V-6		00215	FMP TEE
00035 STD R-2				00125	FMP	Y+3		00216	FBD 12+1
00036 12A	CLA	PXPHI		00126	FBD	12+3		00217	STO TE2
00037 STD R-3				00127	FBD	12+2		00220 49A	CLA TE2
00040 13A	CLA	6-6		00130	FDP	12+1		00221	STO R-20
00041 STD S1G				00131	S1R	COSPSI		00222	50A
148 B55				00132 33A	CLA	COSPSI		00223	F58 X
00047 TSX_PXR				00133	SID	R-15		00224	STO 12+1
00048 15A	TSX	1M6Y, 4		00134 34A	LDO	COSPSI		00225	LDQ Z
00049 TSX_Y				00135	EMP	COSPSI		00226	F5B 12+1
00050 TSX_YTHETA				00136	CMS			00227	STO TEE
00051 TSX_PXPHI				00137	FBD	32+1		00230 51A	LDQ IEB
00052 15A	TSX_Y			00138	B55			00231	FMP 32+5
00053 STD R-4				00139	TSX	SORT, 4		00232	XCA
00054 17A	TSX_Y			00140	STD	VSI		00233	FMP TEE
00055 STD R-5				00141	STO	SIMPsi		00241	STD 12+1
00056 18A	CLA	VTE10		00142	00147 37A	CLA	VSI	00242	LDO TE3
00057 STD R-6				00143 38A	STD	R-17		00243	FMP TE2
00058 19A	CLA	VTE11		00144 38A	STD	R-16		00244	FBD 12+1
00059 STD R-7				00145	FMP	COSPSI		00245	STD R45
00060 20A	CLA	Y		00146	STD	YCI		00246 54A	B55
00061 STD R-8				00147	00153	STD	VCI	00247	TSX SORT, 4
00062 21A	CLA	VTE11		00148 39A	CLA	Z		00250	STD R25
00063 STD R-9				00149 40A	STD	R-18			
00064 22A	CLA	VTE11		00150 40A	CLA	Z			
00065 STD R-10				00151 40A	STD	R-19			
				00152 40A	STD	R-19			
				00153 40A	STD	R-19			
				00154 40A	STD	R-19			
				00155 40A	STD	R-19			
				00156 40A	STD	R-19			
				00157 40A	STD	R-19			

RM 61 TMP-32

00251	53A	CLA R25	00237	F5B TEE	00431	79A	CLA TEE
00252	53A	STO R-22	00240	FAD S1	00432		STO R-32
00253	56A	CLA R25	00241	STO D1	00433	80A	LDQ TEE
	55A		00242	7DQ D1	00434		FMP TEE
00254		TSX SORI,4	00243	STO R-28	00435		STO 12+1
00255		STO R15	00244	CLA X	00436		LDQ TEE
00256	57A	CLS 1E3	00245	F5B 32+7	00437		FMP TEE
00257	57A	STO 1D+1	00246	STO 1D+1	00440		FAD 1D+1
	55A		00247	LDQ 32+7	00441		STO R4H
00260	53A	TSX QATAN,4	00250	FNP 2	00442		CLA R4H
00261		TSX 1E2	00251	XCA			B55
00262		TSX 1D+1	00252	FNP 1D+1			TSX SORI,4
00263		STO THET2S	00253	FAD S2			STO R2M
00264	53A	CLA PI	00254	STO D2			CLA R2M
00265		FSB THET2S	00255	CLA D2			STO R-33
00266	59A1	12E 60A	00256	STO R-29			CLA R2N
00267		TP1 61A	00257	LDQ D2			B55
00270	60A	CLS R15	00258	FNP D2			TSX SORI,4
00271		STO R15	00261	STO 1D+1			STO R1M
00272	61A	LDD 3D+6	00262	LDQ D1			B55
00273		FNP THET2S	00263	FNP D1			TSX QATAN,4
00274		STO THET1S	00264	FAD 1D+1			TSX TE6
00275	62A	CLA R15	00265	STO TE4			00454
00276		STO F-23	00266	7DQ TE4			STO THET2M
00277	63A	CLA THET2S	00267	STO R-30			CLA PI
00300		STO R-24	00270	7DQ 3D+1			FSB THET2M
00301	64A	CLA THET1S	00271	FSB X			T2E 87A
00302		STO R-25	00272	XCA			00461
00303	65A	CLA THET1S	00273	FNP D2			TPL 83A
	55A		00274	STO 1D+2			CLS R1M
00305		TSX SOS,4	00275	LDQ Z			LDQ 3D+6
00306		LDQ SIGN	00277	FAD 1D+2			FMP THET2M
00307		FNP R15	00278	STO 1D+3			STO THET1M
00310		XCA	00401	LDQ X			STO R-34
00311		FNP 1D+1	00402	FNP 3D+7			CLA THET2M
00312		STO S1	00403	FDP TE4			STO R-35
00313	66A	CLA S1	00404				B55
00314		STO R-26	00405	STO TE5			CLA THET1M
00315	67A	CLA THET1S	00406	7DQ			STO R-26
	55A		00407	STO R-31			CLA THET1M
00316		TSX SIN,4	00410	7DQ			B55
00317		STO 1D+1	00411	FSB X			TSX C05,4
00320		LDD SIGN	00412	XCA			STO 1D+1
00321		FNP R15	00413	FNP DJ			LDQ R1M
00322		XCA	00414	STO TEC			FMP 12D+1
00323		FNP 1D+1	00415	LDQ Z			STO AM1
00324		STO S2	00416	FNP D2			00450
00326		STO R-27	00420	FAD TEC			00451
	59A		00421	STO 1D+1			00452
00327	69A	LDQ Z	00422	LDQ X			00453
00330		FNP 2	00423	FNP 3D+7			00454
00332		CNS X	00424	FDP TE4			FMP 1D+1
00333		F5B 3D+1	00425	FNP 1D+1			STO AM2
00334		STO 1D+1	00426	CHS			CLA AM2
00335		LDQ 3D+1	00427	FAD 3D+1			STO R-38
00336		FNP 1D+1	00430	STO TE6			CLA AM1

00516	STO Y	00610	XCB	00702	L00 A4
00517	CLS RM-2	00611	FIP S1	00703	FMP AS
00520	STO Y ₁	00612	F5B 12+1	00704	ST0 12+2
00521	STO AM-2	00613	STO TEG	00705	L00 3D+7
00522	L00 AM-2	00614	CLA .95	00706	FMP X
00523	FIP D2	00615	FDP TE9	00707	FSB 3D+1
00524	STO 1+1	00616	FIP TEG	00710	FSB 1D+2
00525	L00 AM1	00617	FAD 3D+1	00711	FAD 1D+1
00526	F5B 12+1	00620	STO .95	00712	ST0 TEJ
00527	STO TE7	00621	113A	00713	L00 A4
00528	CLA IE7	00622	CLA R4	00714	FMP B5
00529	STO P-39	00623	CLA 3D+1	00715	ST0 1D+1
00530	STO P-39	00624	E5B X	00716	L00 B4
00531	0008A	00625	XCB	00717	FMP AS
00532	L00 B52	00626	FIP S2	00720	FAD Z
00533	FIP D1	00627	STO 1D+2	00721	FAD 1D+1
00534	STO 12+1	00630	L00 Z	00722	ST0 TEK
00535	L00 AM1	00631	FIP S1	00723	L00 B0
00536	FIP D2	00632	FBD 12+2	00724	FMP TEK
00537	FAD 1D+1	00633	FDP TE9	00725	ST0 1D+1
00540	STO TES	00634	STO TEH	00726	L00 A0
00541	101A	00635	STO TEN	00727	FMP TEJ
00542	STO R-40	00636	FAD 2	00730	FAD 1D+1
00543	102A	00635	113A	00731	ST0 PNPX
00544	F5B IE7	00636	FIP .95	00732	124A
00545	STO TIE	00637	STO B4	00733	ST0 R-48
00546	103B	00638	006+1 116A	00734	L00 SINPSI
00547	FIP TES	00639	CLA B4	00735	FMP SINPSI
00550	ST0 TEE	00632	STQ R-45	00736	ST0 Q5
00551	104A	00633	117A	00737	L00 COPSPI
00552	F5B TEE	00634	L00 Z	00738	00240
00553	STO 1D+1	00635	FIP D2	00739	E5B COPSPI
00554	CLA IE7	00636	EDO TEC	00741	XCA
00555	FDP 1D+1	00637	STO 1D+1	00742	FMP 3D+7
00556	ST0 RQ	00638	L00 X	00743	ST0 Q6
00557	105A	00639	FIP 3D+7	00744	J27B
00558	STO R-41	00640	FDP TE4	00745	L00 Q6
00559	106A	00641	STP 1D+1	00746	FMP TE1
00562	ED2 IE7	00642	STO R5	00747	ST0 1D+1
00563	FIP R0	00643	CLA R5	00750	L00 TEE
00564	ST0 BQ	00644	STD R-46	00751	FAD 1D+1
00565	107A	00645	119A	00752	ST0 TE10
00566	ST0 R-42	00646	F5B X	00753	128A
00567	108A	00647	XCB	00754	CLA TE10
00570	FIP S2	00648	FIP D2	00755	ST0 R-49
00571	STO 1D+1	00649	STO 1D+2	00756	L00 3D+7
00572	L00 S1	00650	L00 Z	00757	FMP TEB
00573	FIP S1	00651	FIP O1	00758	ST0 TEB2
00574	F5B 12+1	00652	FDP JE4	00759	J27C
00575	STO TE9	00653	FIP 1D+3	00760	L00 TEB2
00576	109A	00654	STQ .95	00761	FMP Q6
00577	ST0 R-43	00655	L00 X	00762	ST0 TE11
00578	J109 J108	00656	FIP D7	00763	131A
00579	J27C	00657	FIP TE4	00764	CLA TE11
00580	FIP TEE	00658	FIP 1D+3	00765	J27B
00581	F5B TEA	00659	STO 1D+3	00766	FMP S2
00582	ST0 R5	00660	STQ .95	00767	ST0 1D+1
00583	111A	00661	00673	00770	L00 TE10
00584	F5B S2	00662	00674	00771	FMP S1
00585	ST0 1D+1	00663	00675	00772	FSB 1D+1
00586	112A	00664	00676	00773	FDP TE9
00587	S1A S2+1	00665	00677	00774	ST0 1D+1

RM 61TMP-32

00774	STO R6	01066	STO R-54	01160	STO R8
00775	CLA R6	01067	LDA TEH	01161	CLA R8
00776	STO R-51	01070	FNP TER	01162	STO R-57
00777	LDA TE11	01071	FAD Z	01163	CLA 3D+1
01000	FNP S1	01072	STO 12+1	01164	FAD X
01001	STO 13+1	01073	LDA 32+7	01165	XCA
01002	LDA TE10	01074	FNP 12+1	01166	FNP Z
01003	FNP 32	01075	STO 87	01167	XCA
01004	FAD 12+1	01076	CLA 87	01170	FNP S1
01005	FDP YE9	01077	STO R-55	01171	STO 13+2
01006	STO B6	01100	LDA 85	01172	LDA 52
01007	CLA B6	01101	FNP 87	01173	FNP Q3
01010	STO R-52	01102	STO 12+1	01174	FAD 12+2
01011	LDA B5	01103	LDA R5	01175	STO 36
01012	FNP A0	01104	FNP A7	01176	CLA 35
01013	STO 13+1	01105	CHS	01177	FDP TE9
01014	LDA B0	01106	FAD X	01200	FNP 96
01015	FNP A5	01107	FAD 12+1	01201	STO 13+1
01016	FAD 12+1	01110	STO Q1	01202	CLA 3D+7
01017	STO Q1	01111	LDA B5	01203	FAD X
01020	CLA B0	01112	FNP A7	01204	STO 12+2
01021	FNP B5	01113	STO 12+1	01205	LDA Z
01022	STO 12+1	01114	LDA 85	01206	FNP 12+2
01023	LDA R0	01115	FNP B7	01207	FAD 12+1
01024	FNP A5	01116	FAD 12+1	01210	STO B3
01025	FNP 13+1	01117	STO Q2	01211	157A
01026	STO Q2	01120	LDA R0	01212	STO R-58
01027	LDA SINPST	01121	FNP Q2	01213	158A
01030	FNP SINPSI	01122	STO 12+1	01214	FNP B9
01031	CHS	01123	LDA B0	01215	STO 1D+1
01032	FAD R6	01124	FNP Q1	01216	LOA B5
01033	STO Q3	01125	FSP 13+1	01217	FNP A8
01034	CLA R6	01126	STO PNPZ	01220	STO 12+2
01035	FNP Q3	01127	150A	01221	LOA X
01036	STO R6	01130	STO R-56	01222	FNP 32+7
01037	LDA Y	01131	LDA X	01223	XCA
01040	FNP Q1	01132	FNP X	01224	FNP X
01041	XCA	01133	CHS	01225	F5B 1D+2
01042	FNP R6	01134	FAD 3D+1	01226	FAD 12+1
01043	STO 12+1	01135	STO Q3	01227	STO Q1
01044	LDA Y	01136	CLA 3D+1	01228	B55
01045	FNP Q4	01137	FSP X	01230	TEX QW, 4
01046	FSP 12+1	01140	XCA	01231	TEX B5
01047	STO PNPY	01141	FNP S2	01232	TEX B8
01050	CLA R9	01142	XCA	01233	STO 1D+2
01051	STO R-53	01143	FNP Z	01234	B55
01052	LDA 32+7	01144	STO 12+2	01235	TEX Q5
01053	FNP TE9	01145	LDA S1	01236	TEX B8
01054	STO R5	01146	FSP Q3	01237	STO 1D+2
01055	CLA Q5	01147	FSP 1D+2	01238	B55
01056	FDP T13	01150	STO Q4	01239	TEX QW, 4
01057	FNP TS	01151	CLA TEA	01240	162A
01058	STO 12+1	01152	FDP JE2	01241	TEX X
01059	CLA 3D+7	01153	FNP Q4	01242	TSX X
01060	FSP X	01154	FSP 32+1	01243	STO 1D+3
01061	FAD 12+1	01155	STO 12+1	01244	LOQ 3D+7
01062	STO R7	01156	LDA 32+7	01245	FNP 12+3
01063	CLA R7	01157	FNP 1D+1	01246	F5B 1D+2

01247	FAD 12+1		01337	STO R-60	01431	STO 12+1
01250	STO 01		01340	173A	LDD 3)+4	01432 LDD Q1
01251	163A	LDD 85	01341	FMP TEB		01433 FMP 12+1
01252	FMP 98		01342	STO B1		01434 CHS
01253	STO 12+1		01343	174B	CLA B1	01435 STO M-2
01254	LDD 95		01344	STO R-61		01436 LDD W
01255	FMP 88		01345	175A	LDD B1	01437 FMP Y
01256	STO 12+2		01346	FMP S2		01440 STO TAG
01257	LDD X		01347	STO 12+1		01441 LDD TAG
01258	FMP Z		01350	LDD A1		01442 FMP VSI
01259	FBD 12+2		01351	FMP S1		01443 STO IE12
01260	FBD 13+1		01352	FMP 12+1		01444 CLA TE12
01261	FBD 13+1		01353	FMP TE2		01445 STO R-64
01262	FBD 13+1		01354	STO 12+2		01446 LDD U-6
01263	STO 02		01355	CLA 32+1		01447 FMP YCI
01264	164A	LDD 80	01356	FMP 12+2		01450 STO XI
01265	FMP 92		01357	STO A2		01451 LDD W
01266	STO 12+1		01358	FMP A2		01452 FMP QR
01267	LDD 30		01359	STO R-62		01453 STO X2
01268	FMP 01		01360	176A	CLA A2	01454 STO X1
01269	FBD 12+1		01361	STO R-62		01455 FSP X2
01270	FDP 4-2		01362	177A	LDD A1	01456 STO X3
01271	TSX COS-4		01363	FMP S2		01457 CLA W-9
01272	STO Q1		01364	STO 12+1		01458 FDP TE12
01273	XCB		01365	178A	CLA B2	01459 FMP X3
01274	CMS		01366	STO 12+2		01460 LDD W
01275	STO M-12		01367	FMP B1		01461 FMP VCI
01276	165A	CLA 4-4	01368	STO R-63		01462 STO 9-3
	BSS		01369	FDP TE9		01463 LDD U-7
01277	TSX SIN-4		01370	STO B2		01464 FMP YTHE18
01278	STO Q1		01371	179A	CLA B2	01465 STO XI
01279	166A	CLA 4-4	01372	178A	STO TAA	01466 LDD W
01280	BS5		01373	STO B-63		01467 FMP VCI
01281	TSX SIN-4		01374	179A	LDD A2	01468 STO X2
01282	STO Q1		01375	FMP S3		01469 CLA XI
01283	167A	LDD 02	01376	STO TAA		01470 FDP IE12
01284	STO 12+1		01377	180A	LDD B2	01471 FSP X2
01285	FMP 22+4		01378	FMP B5		01472 FMP X3
01286	XCB		01379	STO TAC		01473 LDD W
01287	FMP 92		01380	181A	LDD A2	01474 STO X3
01288	STO 12+1		01381	FMP B5		01475 CLA W-9
01289	LDD Q1		01382	182A	LDD B2	01476 FMP X3
01290	FMP 3D+7		01383	FMP B5		01477 FMP X2
01291	XCB		01384	STO TAC		01478 LDD W
01292	FMP 01		01385	182A	LDD B2	01479 STO X3
01293	FMP 91		01386	FMP S5		01480 CLA XI
01294	FMP 12+1		01387	STO TAA		01481 FMP YCI
01295	STO 03		01388	CLA TAA		01482 FSP X2
01296	STO Q1		01389	183A	CLA TAC	01483 STO X3
01297	168A	CLA 22+1	01390	184A	CLA TAC	01484 LDD W
01298	STO 03		01391	184A	STO 12+1	01485 FMP VR
01299	170A	CLA 22+1	01392	FMP 01		01486 STO 12+1
01300	STO 03		01393	184A	FMP YSI	01487 LDD V-6
01301	170A	CLA 22+1	01394	184A	STO TAC	01488 FMP YTHE18
01302	S2		01395	184A	STO 12+1	01489 FSP 12+1
01303	TSY JU1.2		01396	FMP 01		01490 LDD V-6
01304	CLB TE13		01397	184A	FMP 12+1	01491 FMP X3
01305	TSY JU1.2		01398	184A	STO TAC	01492 STO 6-5
01306	CLB TE13		01399	184A	STO TAC	01493 LDD U-7
01307	TSY JU1.2		01400	184A	STO TAC	01494 LDD V-7
01308	CLB TE13		01401	184A	STO TAC	01495 LDD W
01309	TSY JU1.2		01402	184A	STO TAC	01496 FMP YCI
01310	CLB TE13		01403	184A	STO TAC	01497 FSP X2
01311	TSY JU1.2		01404	184A	STO TAC	01498 STO X3
01312	FMP 3D+7		01405	182A	STO TAC	01499 LDD W
01313	FMP 01		01406	182A	STO TAC	01500 FMP YCI
01314	FMP 91		01407	182A	STO TAC	01501 FSP X2
01315	FMP 12+1		01408	182A	STO TAC	01502 STO X3
01316	STO 03		01409	183A	STO TAC	01503 LDD W
01317	169A	CLA 22+1	01410	183A	STO TAC	01504 FMP VR
01318	STO 03		01411	183A	STO TAC	01505 STO X2
01319	171A	CLA 22+1	01412	183A	STO TAC	01506 LDD V-8
01320	S2		01413	183A	STO TAC	01507 FMP YCI
01321	TSY JU1.2		01421	183A	STO TAC	01508 FSP X2
01322	169A	CLA 22+1	01422	183A	STO TAC	01509 STO X3
01323	TSY JU1.2		01423	183A	STO TAC	01510 LDD W
01324	170A	CLA 22+1	01424	183A	STO TAC	01511 FMP VR
01325	S2		01425	183A	STO TAC	01512 STO 12+1
01326	TSY JU1.2		01426	183A	STO TAC	01513 LDD V-6
01327	170A	CLA 22+1	01427	183A	STO TAC	01514 FMP YTHE18
01328	TSY JU1.2		01428	183A	STO TAC	01515 FSP 12+1
01329	171A	CLA 22+1	01429	183A	STO TAC	01516 LDD V-6
01330	TSY JU1.2		01430	183A	STO TAC	01517 FMP YTHE18
01331	171A	CLA 22+1	01431	183A	STO TAC	01518 FSP 12+1
01332	171A	CLA 22+1	01432	183A	STO TAC	01519 LDD V-6
01333	FPC 111		01433	183A	STO TAC	01520 FMP YTHE18
01334	FPC 111		01434	183A	STO TAC	01521 FSP 12+1
01335	STO 01		01435	183A	STO TAC	01522 LDD V-6
01336	172A	CLA 22+1	01436	183A	STO TAC	01523 FSP 12+1

RM 61 TMP-32

01523	STO X1	01615	F90 12+2	01706	TRB 2428
01524	203A	L08 V51	FAD 13+1	01710	231A CLA V-69
01525	F90	V51	FAD V-8	01710	FSB WIST
01526	STO X2	01616	STO V-5	01710	FSB WIST
01527	204A	CLA X1	01620 212A	01711	231A T2E 2328
01528	FDP X2	01621 213A	CLA 32+3	01712	TPL 2328
01529	STO X3	01622	STO V-3	01713	TRA 2428
01530	L08 X3	01623 214A	CLA 32+3	01714	CLA WIST
01531	F90	13+3	STO V-4	01715	FAD V-62
01532	205A	L08 X3	01624	STO V-4	STD WIST
01533	F90	13+3	01625 215A	CLA 32+3	STD WIST
01534	STO PP51PT	01626	STO V-5	01717	CAL 23+1
01535	206A	CLA PP21PT	01627 216A	CLA TE13	BS5
01536	STO F-5	01628	PP51PT	01720	TSX CSTHD .4
01537	207A	CLA 32+3	01631 217A	TRB 207A	P2E 8240
01538	STO PP51PT	01632 218A	L08 V-6	01722	LXO 23+2,1
01539	208A	CLA 32+3	01633	F90 V-6	01723 235A L08 R+,1
01540	STO PP51PT	01634	STO X1	01724	STR
01541	209A	CLA 32+3	01635 219A	L08 V-7	TXI **1,1,1
01542	F90	13+3	01636	F90 V-7	01726 235A2 TXL 235A,1,69
01543	209A	L08 V-9	01637	STO X2	BS5
01544	F90	13+3	01640 220A	L08 V-8	01727 234A LXO 23+2,1
01545	STO 12+1	01641	F90 X3	01730 238A TRB 2428	TSX (FLD) .4
01546	L08 PP2	01642	STO X3	01731 239A CLA V-69	STD WIST
01547	F90	PPX	01643 221A	CLA X1	F5B 32+1
01548	STO 12+2	01644	FAD X2	01733 239A1 T2E 231A	LXO S1
01549	L08 PP2	01645	FAD X3	01734	TPL 231A
01550	F90	PPX	01646	B55	TSX (FLD) .4
01551	L08 PPV	01647	STO R-66	01735 240A CLA V-62	TRB 2428
01552	F90	PPV	01650 222A	CLA PP51PT	CLA V-69
01553	STO 12+3	01651	STO R-66	01736	STD WIST
01554	L08 PPX	01652 223A	CLA PP51PT	01737 241A TPL 233A	LXO S1
01555	F90	PPX	01653	STO R-67	LXO S2+4
01556	FAD 13+3	01654 224A	CLA TE14	01738	STD WIST
01557	F90	12+2	01655 224A	STO R-68	01739 242A LXO S1
01560	FAD 13+1	01656 224A	CLA V-5	01741	LXO S1+1,2
01561	STO V-6	01657	FDP TE14	01742	LXO S1+1,2
01562	L08 V-9	01660	FDP Y	01743	TRB 1,4
01563	STO 12+2	01661	STO V-6	01744 22	OCT +00000020000000
01564	F90 13+1	01662 226A	CLA V-7	01745	OCT +00000060000000
01565	L08 PPZ	01663 226A	FDP TE14	01746	OCT +000001000000
01566	F90 P2THET	01664 226A	FDP Y	01747	OCT +202622077326
01567	STO 12+2	01665	STO V-7	01748	OCT +204000000000
01568	L08 PPV	01666	FDP TE14	01750	OCT +105447113564
01569	F90 P2THET	01667 226A	FDP TE14	01751	OCT +113715126246
01570	STO 12+3	01668 226A	FDP Y	01752	OCT +000000000000
01571	F90 P2THET	01669 226A	FDP TE14	01753	OCT +203400000000
01572	STO 12+3	01670 226A	FDP Y	01754	OCT +204400000000
01573	L08 PPX	01671 226A	STO V-8	01755	OCT +200400000000
01574	F90 P2THET	01672 226A	STO V-8	01756	OCT +202400000000
01575	ESQ 12+3	01673 228A	STO V-8	01757	OCT +113715126246
01576	F90 12+2	01674 228A	FDP TE14	01758	OCT +000000000000
01577	F90 12+2	01675 228A	FDP Y	01759	OCT +233000000000
01578	F90 V-7	01676 228A	STO V-8	01760	OCT +000000077777
01579	F90 V-7	01677 228A	CLA V-6,7	01761	OCT +000000000000
01580	F90 PP51PT	01678 228A	T2E 239A	01762	OCT +000000000000
01581	STO 12+1	01679 228A	T2E 239A	01763	OCT +000001000000
01582	F90 PPV	01680 228A	TPL 2428	01764	OCT +000000000000
01583	F90 PPV	01681 228A	TPL 233A	01765	BCD 16)
01584	L08 PPX	01682 230A	CLA V-3	01766	BCD 1PE14,
01585	F90 PPX	01683 230A	F98 V-67	01767	BCD 1CH0,1
01586	STO 12+3	01684 230A	F98 V-67	01768	BCD 1CH0,1
01587	L08 PPV	01685 230A	TPL 233A	01769	BCD 1CH0,1
01588	F90 PPV	01686 230A	TPL 233A	01770	BCD 1CH0,1

I. SUBROUTINE ELECTX

The whole process of determining the free normalized electron density and its spatial derivatives at any point in space is explored in this subroutine. For the case under consideration this is a simple sphere where the values in Equation 74 are represented by the following components of the W vector

$$W(28) = R; \quad W(34) = A; \quad W(35) = n$$

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C SIMPLE SPHERE ELECTX GIVEN BY NE=RANGE=M DEC.22,1960 IBM-7090
C WITH INNER SPHERE HAVING NE = CONSTANT FROM RADII = M(36) KM
1 SUBROUTINE ELECTX CX,PXR,PXTHET,PXPHI1
2 DIMENSION RECORD(C125),UC115,M(250)
3 COMMON RECORD,V,W,N
4 CALL BIG
5 TER1 = S(C(342)/W(32))/W(32)*3, 184983E9
6 IF(C(28)+J(36)) 56, 11, 11
7 X = TER1/(NC(36))*W(352)
8 PXR = 0.0
9 PXTHET = 0.0
10 PXPHI = 0.0
11 GO TO 40
12 ST = SIN(W(35))
13 CT = COS(W(35))
14 SP = SIN(W(62))
15 CP = COS(W(62))
16 RST = V(5)*ST
17 RSTCP = RST*CP
18 RSTP = RST*SP
19 RCT = V(6)*ACT
20 XSRP = (W(33)-RSTCP)
21 ZSRP = (W(33)-RCT)
22 PARTR = W(35)*CST+CP*XSRP+ST*SP*YBRP+CT*ZBRP
23 PARTT = W(35)*CRCT+CXBRP*CP+YBRP*SP-RST*ZBRP
24 PARTP = W(35)*CYRP+RSTCP--XBRP*RSTP,
25 TEP2 = W(25)*M(35)
26 X = TER1/TER2
27 TR3 = Y/C(26)**2
28 PXR = TR3*ZSRP
29 PXTHET = TR3*PARTT
30 PXPHI = TR3*PARTP
31 UC36 = UC35*50RTFCX3
32 UC39 = UC39*50RTFCX3
33 UC32 = UC32*50RTFCX3
34 UC392 = UC395*32/3-184283E9*W(32)
35 RETURN
36 END,0,1,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0

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STORAGE LOCATIONS FOR VARIABLES APPEARING IN COMMON STATEMENTS

	DEC	OCT	DEC	OCT	DEC	OCT	DEC	OCT
N 32100 76679			RECORD	32561 77461	V 32549 77445		V 32438 77266	

EXTERNAL FORMULA NUMBERS WITH CORRESPONDING INTERNAL FORMULA NUMBERS AND OCTAL LOCATIONS

EFN	IFN	LOC	EFN	IFN	LOC	EFN	IFN	LOC
1	00000	3	6	00034	5	00042	6	00046
8	00051	9	11	00061	11	00064	12	00067
14	00075	15	17	00100	16	00103	17	00106
12	00114	20	22	00117	21	00122	22	00125
24	00170	26	27	00202	27	00206	28	00211
30	00222	31	32	00225	40	00230	33	00239
						41	00236	42
								35 00244

STORAGE NOT USED BY PROGRAM

	DEC	OCT	DEC	OCT
198	00306		32187 76673	

LOCATIONS OF NAMES IN TRANSFER VECTOR

	DEC	OCT	DEC	OCT	DEC	OCT	DEC	OCT
EXC3	1 00001	COS	3 00003	BIGR	0	00000	SIN	2 00002

STORAGE LOCATIONS FOR VARIABLES NOT APPEARING IN DIVERGENCE, EQUIVALENCE, OR COMMON SENTENCES

	DEC	OCT	DEC	OCT	DEC	OCT	DEC	OCT
CT	197 00305	CP	196 00304	PARTP	195 00303	PARTP	194 00302	PART
SCI	192 00300	STCP	191 00227	EST	190 00226	ESTSP	189 00275	SP
ST	197 00273	TERI	186 00272	TER2	185 00271	TR3	184 00270	XBRP
YNE	182 00264	ZNEP	181 00265					

STORAGE LOCATIONS FOR SYMBOLS NOT APPEARING IN SOURCE PROGRAM

	DEC	OCT	DEC	OCT	DEC	OCT	DEC	OCT
13	176 00260	22	168 00250	37	169 00251	62	171 00253	

ENTRY POINTS TO SUBROUTINES NOT OUTPUT FROM LIBRARY.

B168 B223 S169 C05 S071

RM 61TMP-32

0000000	DIGR	BCD 1BIGR		00067 148	CLA V-4	00156	FMP CP
0000010	EXP C3	BCD 1EXP C3		00070	BSS COS, 4	00157	FAD 12+2
0000020	SIN	BCD 1SIN		00071	TSX C1	00160	STO 12+3
0000030	COS	BCD 1C05		00072	STO C1	00161	LDQ RCT
0000040	SQRT	BCD 1SQRT		00073	CLA V-5	00162	FMP 12+3
0000050	\$	HTR		00074	BSS	00163	F5B 12+1
0000060		HTR		00075	TSX SIN, 4	00164	STO 12+4
0000070		HTR		00076*	STO SP	00165	LDQ 4-34
0000080		BCD 1ELECTX		00075 169	CLA V-5	00166	FMP 12+4
0000090		SXD \$, 1		00077	BSS	00167	STO PARTT
0000100		SXD \$+1, 2		00078	TSX COS, 4	00170	LDQ XBRP
0000110		SXD \$+2, 4		00079	STO CP	00171	FMP RSTSP
0000120		CLA 1, 4		00080	LDO V-3	00172	STO 12+1
0000130		STA SR+17		00081	FIP ST	00173	LDQ YBRP
0000140		STA SR+109		00082	STO RST	00174	FMP RSTCP
0000150		STA SR+113		00083	LDO RST	00175	F5B 12+1
0000160		STA SR+125		00084	FIP CP	00176	STO 12+2
0000170		STA SR+131		00085	STO RSTCP	00177	LDQ 4-34
0000180		CLA 2, 4		00086	LDO RST	00200	FMP 12+2
0000190		STA SR+19		00087	FIP SP	00201	STO PARTP
0000200		STA SR+118		00088	STO RSTSP	00202	CLA 4-27
0000210		CLA 3, 4		00089	LDO V-3	00203	LDQ 4-34
0000220		STA SR+21		00090	FIP CT	00204	BSS
0000230		STA SR+121		00091	STO RCT	00205	TSX EXP C3, 4
0000240		CLA 4, 4		00092	CLA V-30	00206	STO TER1
0000250		STA SR+23		00093	F5B RSTCP	00207	FDP TER2
0000260		STA SR+124		00094	STO XBRP	00210	STO X
0000270		5A 655		00095	CLA V-31	00211	LDQ 4-27
0000280		TSX BIGR, 4		00096	F5B RSTSP	00212	FMP 4-27
0000290		CLA 6-33		00097	STO VBRP	00213	STO 12+1
0000300		FDP V-2		00098	CLA V-32	00214	CLA X
0000310		XCR		00099	F5B RCT	00215	FDP 12+1
0000320		000327	FDP N-2	00100	STO ZBRP	00216	STO TR3
0000330		FDP 35		00101	LDO CT	00217	LDQ TR3
0000340		STA JEP1		00102	FIP ZBRP	00218	FMP PARTR
0000350		CLA V-27		00103	STO 12+1	00220	FMP PARTR
0000360		F5B N-35		00104	LDO YBRP	00221	STO PXR
0000370		00044 7A1	TZE 130	00105	FIP ST	00222	31 A
0000380		JEP1 JEP2		00106	XCR	00223	FMP PARTR
0000390		CLA V-33		00107	FIP SP	00224	STO PXHET
0000400		JEP2 N-35		00108	STO 12+2	00225	LDQ TR3
0000410		LDQ N-35		00109	LDO XBRP	00226	FMP PARTR
0000420		555		00110	FIP SI	00227	STO PXHET
0000430		TSX EXP C3, 4		00111	CLA V-34	00230	BSS
0000440		STA 121		00112	XCR	00231	TSX SQRT, 4
0000450		5A 121		00113	FIP CP	00232	STO 12+1
0000460		CLA V-35		00114	FAD 12+2	00233	LDQ 4-2
0000470		LDQ N-35		00115	LDQ RST	00237	FDP 35
0000480		555		00116	E5B ZBRP	00240	EMP 4-2
0000490		TSX EXP C3, 4		00117	STO 12+1	00241	XCR
0000500		STA 121		00118	FIP CP	00242	FMP 4-2
0000510		5A 121		00119	STO SP	00243	STO 4-38
0000520		CLA V-4		00120	LDQ YBRP	00244	LDQ 5, 1
0000530		FDP 12+1		00121	FIP SP	00245	LDQ S+1, 2
0000540		5T9 X-31		00122	STO PARIB		
0000550		CLA 3, 1		00123	F5B 12+1		
0000560		STA PARIB		00124	LDQ 12+3		
0000570		5A 339		00125	STO 12+1		
0000580		TSX SU1, 4		00126	FIP SP		
0000590		555		00127	LDQ SP		
0000600		TSX SU1, 4		00128	STO 12+2		
0000610		STA ST		00129	LDQ XBRP		

J. SUBROUTINE BIGR

This subroutine is used to calculate the distance from the center of the ionizing source to the spatial point r, θ, Ψ at which the electron density and spatial gradients are desired. For the simple sphere this distance is $W(28) = \mathcal{R}$. In addition to this it calculates the angle that \mathcal{R} makes with respect to the vertical distance passing through the center of the ionizing source.

STORAGE LOCATIONS FOR VARIABLES APPEARING IN COMMON STATEMENTS

	DEC	OCT	LOC	DEC	OCT	LOC	DEC	OCT	LOC
N 32138 76674	REC	REC	00000	REC	OCT	00021	REC	OCT	00027
RECORD 32561 77455	V	V	00013	REC	OCT	00070	REC	OCT	00136
			00043	REC	OCT	00070	REC	OCT	00136
			00145	REC	OCT	00146	REC	OCT	00177
			00146	REC	OCT	00146	REC	OCT	00177
			-20 01202	REC	OCT	-20 01202	REC	OCT	-20 01202

STORAGE NOT USED BY PROGRAM

	DEC	OCT	LOC	DEC	OCT	LOC
135 00233	REC	OCT	32187 76673			

LOCATIONS OF VARS IN TRANSFER VECTOR

	DEC	OCT	LOC	DEC	OCT	LOC	DEC	OCT	LOC
COS 1 00201	ARCS	OCT	00003	SIN	OCT	00000	SQRT	OCT	00002

STORAGE LOCATIONS FOR VARIABLES NOT APPEARING IN DIMENSION, EQUIVALENCE, OR COMMON SENTENCES

	DEC	OCT	LOC	DEC	OCT	LOC	DEC	OCT	LOC
RCT 134 00232	RSTCP	OCT	00231	RST	OCT	00230	RSTSP	OCT	00227
TER1 143 00235	TERM1	OCT	00231	TER2	OCT	00228	TER2	OCT	00226

STORAGE LOCATIONS FOR SYMBOLS NOT APPEARING IN SOURCE PROGRAM

	DEC	OCT	LOC	DEC	OCT	LOC	DEC	OCT	LOC
144 00220	2D	OCT	00206	3D	OCT	00207	6D	OCT	00213

ENTRY POINTS TO SUBROUTINES NOT OUTPUT FROM LIBRARY.

	SIN	COS	SORT	ACOS
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RM 61 TMP-32

00000000	SIN	B00 13IN		BSS	00156	FMP W-54
00000001	COS	B00 1005	00066	TSX SORT,4	00157	STO 12+2
00000002	SQRT	B00 13ORT	00167	STO 9-27	00160	LDD 4-18
00000003	ACOS	B00 1ARCOS	00079	CLA 6-53	00161	FMP 4-18
00000004	HTR		00071	FSB RCT	00162	STO 12+3
00000005	HTR		00072	STO 13	00163	LDD 4-3
00000006	HTR		00073	LDD 12	00164	FMP Y-3
00000007	HTR	1BTGF	00074	FMP 12	00165	FSB 12+3
00000008		5X0 5-1	00075	STO 12+1	00166	FSB 12+2
00000009		5X0 5+1,2	00076	CLA 6-52	00167	FDP 1D+1
00000010		5X0 5+2,4	00077	F3B RSTSP	00170	STQ TE2
00000011		5X0 5-4	00100	STO 13	00171	17A
00000012		BS5	00101	LDD 12	00172	FMP TE2
00000013		T3N 3IN,4	00102	FMP 12	00173	CHS
00000014		STD 12+1	00103	STO 12+2	00174	FAD 3D+3
00000015		LDD 12-3	00114	CLA 6-51	00175	BSS
00000016		FMP 12+1	00115	F3B RSTOP	00176	TSX SORT,4
00000017		STD PSET	00116	STO 12	00176	STO TE3
00000018		CL4 U-5	00137	LDD 12	00177	18A
00000019		F3B	00110	FMP 12	00177	TSX ARCS5,4
00000020		T3N 3C5,4	00111	FAD 12+2	00200	TSX TE3
00000021		STD 12+1	00112	FAD 12-1	00201	STO 4-56
00000022		LDD RST	00113	BSS	00202	LX 5
00000023		FMP 12+1	00113	TSX SORT,4	00203	LX 5+1,2
00000024		STD RSTCP	00114	ST 6-5	00204	LX 5+2-4
00000025		CL4 U-5	00114	LDD 4-29	00205	TRA 1,4
00000026		BS5	00115	FMP 3D	00206	OCT +00000020000000
00000027		T3N 3IN,4	00115	XCA	00207	OCT +024000000000
00000028		STD 12+1	00120	FMP 4-27	00210	OCT +15165376246
00000029		LDD RST	00121	STO 12+1	00211	OCT +000000000000
00000030		FMP 12+1	00122	LDD 6-3	00212	OCT +201400040000
00000031		STD RSTSP	00123	FMP V-3	00213	OCT +233000000000
00000032		CL4 U-4	00124	STO 12+2	00214	OCT +000000077777
00000033		BS5	00125	LDD 4-27	00215	OCT +000000000000
00000034		T3N 3C5,4	00126	FMP 4-27	00216	OCT +00000010000000
00000035		STD 12+1	00127	STO 12+3	00217	OCT +000000000000
00000036		LDD U-3	00130	LDD 4-29		
00000037		FMP 12+1	00131	FMP 4-29		
00000038		STD RCT	00132	FAD 12+3		
00000039		CL4 U-32	00133	FSB 12+2		
00000040		F3B RCT	00134	FDP 12+1		
00000041		STD 13	00135	ST9 TERM1		
00000042		LDD 12	00136	CLA TERM1		
00000043		FMP 12	00137	SSP		
00000044		STD 12+1	00140	F3B 32+1		
00000045		CL4 U-31	00141	12A1		
00000046		F3B RSTSP	00142	TPL 15A		
00000047		STD 13	00143	CLA 3D+2		
00000048		LDD 12	00144	STO 4-28		
00000049		F3B RSTCP	00150	16A		
00000050		STD 13	00151	FMP 3D		
00000051		CL4 U-31	00152	XCA		
00000052		F3B RSTSP	00153	FMP 4-18		
00000053		STD 13	00154	STO 4-54		
00000054		LDD 12	00155	LDD 4-54		
00000055		FMP 12	00156	CL5 TERM1		
00000056		STD 12+2	00157	FMP 4-28		
00000057		CL4 U-30	00158	STO 4-28		
00000058		F3B RSTCP	00159	XCA		
00000059		STD 13	00155	FMP 4-18		
00000060		LDD 12+2	00156	STO 4-54		
00000061		FAD 12+1	00155	LDD 4-54		
00000062		STD 13	00155	CL5 TERM1		
00000063		FMP 12+2	00156	FMP 4-18		
00000064		STD 13+1	00155	LDD 4-54		
00000065		FAD 12+1	00155	CL5 TERM1		

K. SUBROUTINE MAGY

The normalized earth's magnetic field and its spatial derivatives are calculated in this subroutine. In this program it is represented by an assumed magnetic dipole field with the dipole located at the center of the earth.

```

C   SUBROUTINE MAGYC  DECEMBER 22, 1960   IBM-7090
      SUBROUTINE MAGYC(V,VR,VTHETA,VPHI,DYDR,DYOTHE,DYDPHI)
      DIMENSION RECORD(122),WC1112,W(250)
      COMMON RECORD,U,N
      TERM7 = SINFC VCS2
      TERM8 = COSFC VCS2
      TERMS = SQRTFC C1.0 + 3.0* TERM8**2
      TERM0 = WC1112/V(4)**3
      T1 = WC52*TERM10/W(32)
      V = T1 + TERM9
      VR = - 1.0 + T1 * TERMS
      VTHETA = - T1 * TERM7
      VPHI = 0.0
      DYDR = (-3.0+V(4)/C0)
      DYOTHE = (-3.0+V(4)*TERM7+TERM8*(TERM9**2))
      DYDPHI = 0.0
      RETURN
      ENDCC 1.0,1.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0

```

STORAGE LOCATIONS FOR VARIABLES APPEARING IN COMMON STATEMENTS

	DEC	OCT	DEC	OCT	DEC	OCT	DEC	OCT	DEC	OCT
N 32188 76674	RECORD	32561	77461	V 32549 77445	W 32438 77266					

STORAGE NOT USED BY PROGRAM

	DEC	OCT	DEC	OCT	DEC	OCT	DEC	OCT	DEC	OCT
107 00193	32187 76673									

LOCATIONS OF NAMES IN TRANSFER VECTOR

	DEC	OCT	DEC	OCT	DEC	OCT	DEC	OCT	DEC	OCT
COS 1 00001	SIN 0 00000	SQRT 2 00002								

STORAGE LOCATIONS FOR VARIABLES NOT APPEARING IN DIMENSION, EQUIVALENCE OR COMMON SENTENCES

	DEC	OCT	DEC	OCT	DEC	OCT	DEC	OCT	DEC	OCT
Y1 108 00192	TERM10 103 00131	TERM11 104 00150	TERMS 103 00147	TERM9 102 00146						

STORAGE LOCATIONS FOR SYMBOLS NOT APPEARING IN SOURCE PROGRAM

	DEC	OCT	DEC	OCT	DEC	OCT	DEC	OCT	DEC	OCT
12 100 00144	23 89 00131	32 91 00133	62 95 00137							

ENTRY POINTS TO SUBROUTINES NOT OUTPUT FROM LIBRARY.

YIN , COS , SQRT

RM 61TMP-32

00000	SIN	BCD 15IN		000067	FNP 3D+2
00001	COS	BCD 10CS		00070	XCR
00002	SQRT	BCD 15QRT		00071	FNP T1
00003	\$	HTR		00072	CHS
00004		HTR		00073	STO VR
00005		HTR		00074	LDA TI
00006		BCD 1MAGY		00075	FNP TERM7
00007		SQD \$,1		00076	CHS
00010		SQD \$,1,2		00077	STO YTHETA
00011		SQD \$,2,4		00078	CLA 3D+3
00012		CLA 1,4		00101	STO VPHI
00013		STA Y\$+27		00102	LDA 3D+1
00014		STA S\$+1		00103	FNP Y
00015		STA S\$+31		00104	CHS
00016		CLA 2,4		00105	FUP U-3
00017		STA S\$+33		00106	STO DYOR
00020		CLA 3,4		00107	LDO TERM9
00021		STA S\$+37		00110	FNP TER9
00022		CLA 4,4		00111	STO 1D+1
00023		STA S\$+39		00112	LDO TERM8
00024		CLA 5,4		00113	FNP 3D+1
00025		STA S\$+44		00114	XCR
00026		CLA 6,4		00115	FNP Y
00027		STA S\$+56		00116	XCR
00030		CLA 7,4		00117	FNP TERM7
00031		STA Y\$+38		00120	CHS
00032	\$R	CLA U-4		00121	FDP 12+1
		BS5		00122	STQ BDYTHE
00033		TSX S1N,4		00123	CLA 3D+3
00034		STO TERM7		00124	STO DYDPHT
00035	68	CLA Y-4		00125	LXD S,1
		BSS		00126	LXD S+1,2
00036		TSX S05,4		00127	LXD S+2,4
00037		STO TERMS		00130	TRA 8,4
00040	78	LDQ TERMS		00131	OCT +00000000000000
00041		FNP TERMS		00132	OCT +00000000000000
00042		STO 1D+1		00133	OCT +201400000000
00043		LDA 3D+1		00134	OCT +202600000000
00044		FNP 1D+1		00135	OCT +29249999999999
00045		FSD 3D		00136	OCT +00000000000000
		BSS		00137	OCT +23300000000000
00046		TSX S08T,4		00140	OCT +000000077777
00047		STO TERMS		00141	OCT +00000000000000
00050	88	CLA Y-18		00142	OCT +00000000000000
00051		FNP U-3		00143	OCT +00000000000000
00052		STO 1D			
00053		FNP 1D			
00054		LIS 35			
7		FNP 12			
00055		STO 1TERM10			
00057	98	CLA Y-24			
00060		FNP U-2			
00061		FNP TERM10			
00062		STO Y			
00063	108	LDQ T1			
00064		FNP TERMS			
00065		STO Y			
00066	118	LDQ TERMS			

L. SUBROUTINE COLFRZ

The normalized collision frequency and its derivatives at a spatial point are determined in this subroutine. In this presentation it is assumed that collision frequency varies exponentially with height. One should refer to the definition of the W vector where the required coefficients for the height stratifications are defined.

RM 61 TMP-32

STORAGE LOCATIONS FOR VARIABLES APPEARING IN COMMON STATEMENTS

	DEC	OCT	DEC	OCT	DEC	OCT	DEC	OCT
N 32198 76674 RECORD 32561 77461	0	32549 77445	U	32438 77266				

EXTERNAL FORMULA NUMBERS WITH CORRESPONDING INTERNAL FORMULA NUMBERS AND OCTAL LOCATIONS

	EFN	IFN	LOC	EFN	IFN	LOC	EFN	IFN	LOC
1	1 00000	2	00027	3	00033	4	10 00047	5	11 00053
6	12 00055	9	00060	12	00064	10	18 00071	13	19 00075
11	22 00102	14	00106	15	00113	7	29 00120		

STORAGE NOT USED BY PROGRAM

	DEC	OCT	DEC	OCT	DEC	OCT	DEC	OCT
EXP 0 00000 100 00144		32187 76673						

LOCATIONS OF NAMES IN TRANSFER VECTOR

	DEC	OCT	DEC	OCT	DEC	OCT	DEC	OCT

STORAGE LOCATIONS FOR VARIABLES NOT APPEARING IN DIMENSION, EQUIVALENCE OR COMMON SENTENCES

	DEC	OCT	DEC	OCT	DEC	OCT	DEC	OCT
B 99 00143	A	98 00142	TERM1	97 00141				

STORAGE LOCATIONS FOR SYMBOLS NOT APPEARING IN SOURCE PROGRAM

	DEC	OCT	DEC	OCT	DEC	OCT	DEC	OCT
15 99 00137	23	84 00124	33	83 00125	63	90 00132		

ENTRY POINTS TO SUBROUTINES NOT OUTPUT FROM LIBRARY.

EXP

RM 61TMP-32

000000 EXP	- BCD_1EXP	-
000001 S	- HTR	-
000002	HTR	-
000003	HTR	-
000004	BCD 1COLFRZ	-
000005	SXD 5,1	-
000006	SXD 5+1,2	-
000007	SXD 5+2,4	-
000010	CJA 1,4	-
000011	STX 5A+19	-
000012	STX 5A+21	-
000013	STX 5A+57	-
000014	CJA 4,4*	-
000015	STX 5A+23	-
000016	STX 5A+59	-
000017	CJA 5,1*	-
000020	STX 5A+25	-
000021	CJA 4,4	-
000024	STX 5A+27	-
000025	CJA 5,3	-
000026	FBB 5D	-
000027	TZE 5A	-
000028	TPL 149	-
000029	CJA 5-26	-
000030	STO 5,1	-
000031	CJA 5-25	-
000032	STO 5	-
000033	CJA 5-3	-
000034	FBB N-19	-
000035	STO 15+1	-
000036	LOG A	-
000037	FMP 15+1	-
000040	CHS	-
000041	STO TERMINI	-
000042	CJA TERMINI	-
000043	B35	-
000044	TSX EXP,4	-
000045	FOP N-2	-
000046	STO 2	-
000047	L09 A	-
000054	STO P2THET	-
000055	CJA 32+1	-
000056	STO P2PHI	-
000057	TRA 29A	-
000060	CJA 5-3	-
000061	FBB 32+2	-
000062	TZE 15A	-
000063	TPL 189	-
000064	CJA 5-70	-
000065	STO 5,1	-
000066	CJA 5-71	-
000067	TPL 89	-

M. SUBROUTINE RCOORD

This subroutine permits the transformation from the earth centered
geomagnetic spherical coordinate system, to the radar coordinate
system.

STORAGE LOCATIONS FOR VARIABLES APPEARING IN COMMON STATEMENTS

	DEC	OCT	DEC	OCT	DEC	OCT	DEC	OCT
G 32165	76645	N 32188	76674	RECORD	32561	77461	V 32549	77445
XN 32187	76673	VN 32180	76664	ZN	32173	76655		

EXTERNAL FORMULA NUMBERS WITH CORRESPONDING INTERNAL FORMULA NUMBERS AND OCTAL LOCATIONS

	EFN	IFN	LOC	EFN	IFN	LOC	EFN	IFN	LOC		
10	20	00205	19	21	00210	20	23	00215	20	24	00220

STORAGE NOT USED BY PROGRAM

	DEC	OCT
	202	00312
		32156 76634

LOCATIONS OF NAMES IN TRANSFER VECTOR

	DEC	OCT	DEC	OCT	DEC	OCT	DEC	OCT
COS	1	00001	ARCOS	3	00003	SIN	0	00000

STORAGE LOCATIONS FOR VARIABLES NOT APPEARING IN DIMENSION, EQUIVALENCE OR COMMON SENTENCES

	DEC	OCT	DEC	OCT	DEC	OCT	DEC	OCT
ETA	201	00311	ETAN	200	00310	EPS	199	00307
ANGA	196	00304	RCE2	195	00303	RCE	194	00302
YP	191	00277	ZETAN	190	00276	ZETA	189	00275

STORAGE LOCATIONS FOR SYMBOLS NOT APPEARING IN SOURCE PROGRAM

	DEC	OCT	DEC	OCT	DEC	OCT	DEC	OCT
10	182	00266	2>	174	00256	3>	175	00257
							6>	177 00261

ENTRY POINTS TO SUBROUTINES NOT OUTPUT FROM LIBRARY.

SIN	COS
SORT	ARCOS

RM 61 TMP-32

00000 SIN	BCD 1SIN	00066	STO 1Z+5	00160	STO ETA
00001 COS	BCD 1COS	00067	LDQ XP	00161	CLA ZETAN
00002 SQR	BCD 1SQR	00070	FMP XN	00162	FDP ZH-7
00003 ARCS	BCD 1ARCS	00071	FAD 1Z+4	00163	LDQ ZETA
00004 S	HTR	00072	FAD 1Z+5	00164	LDQ ZETA
00005	HTR	00073	FSB 1Z+3	00165	FMP ZETA
00006	HTR	00074	STO EPSN	00166	STO 1Z+1
00007	BCD 1RC00R0	00075	11A	00167	LDQ ETA
00008	SXD S-1	00076	FMP YN-5	00170	FMP ETA
00009	SXD S+1,2	00077	STO 1Z+1	00171	FAD 1Z+1
00010	SXD S+2,4	00078	LDQ XP	00172	STO ROE2
00011	CLA V-4	00079	FMP YN-4	00173	CLA RCE2
00012	BSS	00101	STO 1Z+2	00174	BSS
00013	CLA V-4	00102	LDQ VP	00175	TSX SQRT,4
00014	TSX SIN,4	00103	FMP YN-3	00176	STO RCE
00015	STO 1Z+1	00104	STO 1Z+4	00177	LDQ EPS
00016	LDQ V-3	00105	FAD 1Z+2	00178	FMP EPS
00017	FMP 1Z+1	00106	FAD 1Z+1	00179	FAD RCE2
00020	STO RST	00107	STO 1Z+3	00200	BSS
00021	CLA V-5	00110	LDQ 2P	00201	TSX SQRT,4
00022	BSS	00111	FMP YN-2	00202	STO W-59
00023	TSX COS,4	00112	STO 1Z+4	00203	CLA W-59
00024	STO 1Z+1	00113	LDQ XP	00204	TZE 31A
00025	LDQ RST	00114	FMP YN-1	00205	CLA RCE
00026	FMP 1Z+1	00115	STO 1Z+5	00206	FDP W-59
00027	FSB XN-6	00116	LDQ VP	00207	STO 1Z+1
00028	STO XP	00117	FMP YN	00208	BSS
00030	CLA V-5	00120	FAD 1Z+5	00210	TSX ARCS,4
00031	BSS	00121	LDQ 2P	00211	TSX 1Z+1
00032	TSX SIN,4	00122	FMP ZH-4	00212	STO ANGE
00033	STO 1Z+1	00123	STO ETAN	00213	CLA RCE
00034	LDQ RST	00124	12A	00214	TZE 31A
00035	FMP 1Z+1	00125	LDQ YP	00215	CLA ZETA
00036	FSB VN-6	00126	STO 1Z+1	00216	FDP RCE
00037	STO VP	00127	LDQ 2P	00217	STO 1Z+1
00038	CLA V-4	00130	FMP ZH-4	00218	BSS
00039	BSS	00131	STO 1Z+2	00219	TSX ARCS,4
00040	TSX COS,4	00132	LDQ XP	00220	24A
00041	STO 1Z+1	00133	FMP ZH-3	00221	TSX 1Z+1
00042	LDQ V-3	00134	FAD 1Z+2	00222	STO ANGE
00043	FMP 1Z+1	00135	STO 1Z+3	00223	25A
00044	STO 1Z+1	00136	FAD 1Z+1	00224	FDP 3J
00045	LDQ XP	00137	LDQ XP	00225	STO W-60
00046	10A	00140	FMP ZH-2	00226	LDQ ANGE
00047	FMP XN-5	00141	STO 1Z+4	00227	FDP 3J
00048	STO 1Z+1	00142	LDQ YP	00230	STO W-61
00049	LDQ VP	00143	FMP ZH-1	00231	CLA W-17
00050	FMP ZH-6	00144	STO 1Z+5	00232	F5B ANGE
00051	LDQ 2P	00145	LDQ 2P	00233	XCA
00052	FMP XN-4	00146	FMP ZH	00234	FMP 3J
00053	STO 1Z+2	00147	FAD 1Z+5	00235	STO W-79
00054	LDQ 2P	00148	LDQ YP	00236	STO W-80
00055	FMP XN-3	00149	FAD 1Z+4	00237	F5B ANGE
00056	STO 1Z+2	00150	FSB 1Z+3	00238	XCA
00057	FAD 1Z+1	00151	STO ZETAN	00240	FMP 3J
00058	STO 1Z+3	00152	LDQ VP	00241	FMP 3J
00059	LDQ VP	00153	13A	00242	STO W-80
00060	FMP XN-2	00154	CLA EPSN	00243	CLA V-9
00061	STO 1Z+4	00155	STO EPS	00244	F5B W-59
00062	LDQ 2P	00156	CLA ETAN	00245	STO W-81
00063	STO 1Z+4	00157	FDP ZH-7		

00246	30A	LDD	3D+1
00247		FNP	V-10
00250		FSB	W-59
00251		STO	W-82
00252	31A	LXD	S-1
00253		LXD	S+1,2
00254		LXD	S+2,4
00255		TRA	1,4
00256	22	OCT	+000002000000
00257	3D	OCT	+206712273407
00260		OCT	+233444607400
00261	62	OCT	+233000000000
00262		OCT	+000000077777
00263		OCT	+000000000000
00264		OCT	+000001000000
00265		OCT	+000000000000

N. SUBROUTINE OUTONE

In this presentation this subroutine is used to list the input data and the initial conditions that define the ray tracing problem on Tape Unit 6. Table 5-1 illustrates this output by the statement of the input RECORD and the next ninety-one words of data which define in order the first 70 components of the W vector, followed by the first 21 components of the V vector. This subroutine can be greatly improved.

```
C   SUBROUTINE OUTONE DECEMBER 22, 1960 IBM-7090
1   SUBROUTINE OUTONE (SIGN,10)
2   COMMON RECORD,U,M,N
3   DIMENSION RECORD(125),U(111),W(250)
4   IF (SIGN) 3,3,6
5   WRITE OUTPUT TAPE 6,4,10,RECORD(1),1=1,12
6   FORMAT(25H1 EXTRA-ORDINARY RAY ,16,12A6)
7   GO TO 8
8   WRITE OUTPUT TAPE 6,7,10,RECORD(12),1=1,12
9   FORMAT(22H1 ORDINARY RAY 16,12A6)
10  WRITE OUTPUT TAPE 6,9,CYC(12),1=1,70, CYC(12),1=1,21
11  FORMAT(1H0,14.75
12  RETURN
13  END(1,0,1,0,0,0,0,0,0,0,0,0,0,0)
```

RM 61TMP-32

STATEMENTS OF APPAREL & ACCESSORIES FOR WHICH NO COMMON STATEMENT IS PROVIDED.

INTERNAL FORMULA NUMBERS WITH CORRESPONDING INTERNAL FORMULA NUMBERS AND OCTAL LOCATIONS

LOCATIONS OF NAMES IN TRANSFER VECTOR

STORAGE LOCATIONS FOR SYMBOLS NOT APPEARING IN SOURCE PROGRAM

9 67 00003 007 DEC 001 73 00111 804 DEC 001 80 00120 22 DEC 001 58 00072 60 DEC 001 60 00074

ESTATE PLANNING TO MAXIMIZE THE TAX BENEFITS

0000000000	CSTM2	BCD 1	CSTM2		29A	BSS	
000001	CF1L3	BCD 1	CF1L3		00065	TSX CF1L3,4	
000002	\$	HTR	\$		00066	30B	LxD \$,1
000003	\$	HTR	\$		00067		LxD \$,1,2
000004		HTR			00070		LxD \$,2,4
000005		BCD 1	TONE		00071		TRA 3,4
000006		SXD \$,1			00072	/2	OCT +0000006000000
000007		SXD \$,1,2			00073		OCT +000001000000
000010		SXD \$,2,4			00074	/62	OCT +230000000000
000011		CLA 1,4			00075		OCT +00000077777
000012		STA 88			00076		OCT +000000000000
000013		CLA 2,4			00077		OCT +00001000000
000014		STA 88+6			00100		OCT +000000000000
000015		STA 88+18			00101		BCD 14,7)
000016	88	CLA SIGN			00102		BCD 1 IPBE1
000017	881	TZE 9A			00103	833	BCD 1 C1H0,
000020		TPL 168			00104		BCD 1,12862
000021	9A	CAL 23			00105		BCD 1 -16
		BSS			00106		BCD 1 RAY
000022		TSX CSTM2,4			00107		BCD 1 DINARY
000023		PZC 80			00110		BCD 1 OR
000024	10A	LxD 1D			00111	837	BCD 1 C22H1
000025		STR			00112		BCD 162
000026	11A	LxD 2D+1,1			00113		BCD 116,128
000027	12A	LxD RECORD+1,1			00114		BCD 1 RAY
000030		STR			00115		BCD 1 NARY R
000031	1281	TX1 +*1:1,1			00116		BCD 1-A-ORDI
000032	1282	TX1 128,1,12			00117		BCD 1 EXTR
	148	BSS			00120	824	BCD 1C23H1
000033	138	TRA 228					
000034	138	TRA 228					
000035	16A	CAL 23					
		BSS					
000036		TSX CSTM2,4					
000037		PZC 827					
000040	17A	LxD 1D					
000041		STR					
000042	18A	LxD 2D+1,1					
000043	19A	LxD RECORD+1,1					
000044		STR					
000045	1991	TX1 +*1:1,1					
000046	1992	TX1 198,1,12					
000047		BSS					
000050	228	TSX CSTM2,4					
000051		BSS					
000052		TSX CSTM2,4					
000053	238	PZC 829					
000056	2481	TX1 +*1:1,1					
000057	2482	TX1 248,1,70					
000060	25A	LxD 2D+1,1					
000061	2779	LxD V+1,1					
000062		STR					
000063	2791	TX1 +*1:1,1					
000064	2792	TX1 278,1,21					

O. SUBROUTINE OUTPUT

This subroutine will presently yield the output data on Tape Unit 6 that is summarized in Tables 5 and 6. Following the information that is the result of subroutine OUTONE, Tables 5-1 and 5-2 illustrate the information obtainable after each integration. Beginning with the first word this information has the following meaning:

- Integration number W(70)
- Vector components V(2), V(3)
- Height above earth surface (km)
- Angle θ in degrees
- Angle Ψ in degrees
- Vector components V(7) through V(21)
- Vector components W(1) through W(11)
- Vector components W(28), W(29), W(38), W(39)
- Vector components W(55) through W(70)
- Vector components W(80) through W(90)

Under certain conditions of vector components W(68) and W(69) the R vector described in subroutine RINDEX will be listed between each of these sets of data for each time that the RINDEX subroutine is entered.

In addition to this data, the data summarized in Table 6 is listed on Tape Unit 10.

STORAGE LOCATIONS FOR VARIABLES APPEARING IN COMMON STATEMENTS.

	DEC	OCT	DEC	OCT	DEC	OCT	DEC	OCT
G 32165 76645			N 32168 76674		RECORD 32561 77461		V 32549 77445	
IN 32187 76673			M 32180 76664		ZN 32173 76655		W 32438 77266	

EXTERNAL FORMULA NUMBERS WITH CORRESPONDING INTERNAL FORMULA NUMBERS AND OCTAL LOCATIONS.

EFN	IFN	LOC	EFN	IFN	LOC	EFN	IFN	LOC	EFN	IFN	LOC		
1	00000	9	5	00000	50	6	00000	11	7	00000	12	8	00000
32	9 00000	3	13	00016	4	14	00020	5	17	00047	6	18	00052
7	23 00074	7	24	00077	9	25	00124	9	40	00163	10	42	00167
10	46 00177	30	53	00215	31	54	00220	31	55	00223	33	57	00253
40	59 00255												

STORAGE NOT USED BY PROGRAM

	DEC	OCT	DEC	OCT	DEC	OCT
	234	00352			32156	76634

LOCATIONS OF NAMES IN TRANSFER VECTOR

	DEC	OCT	DEC	OCT	DEC	OCT
RECORD	0 00000	CFILE 2 00002			CSTM 1 00001	

STORAGE LOCATIONS FOR VARIABLES NOT APPEARING IN DIMENSION, EQUIVALENCE OR COMMON SENTENCES

	DEC	OCT	DEC	OCT	DEC	OCT	DEC	OCT
IN	233 00351	11 232 00350			TE1 231 00347		TE2 230 00346	

STORAGE LOCATIONS FOR SYMBOLS NOT APPEARING IN SOURCE PROGRAM

	DEC	OCT	DEC	OCT	DEC	OCT	DEC	OCT
REC	011 222 00336	910 197 00305	80C 3D	218 00332	80B 221	00335	808 225	00341
	12 226 00342	20 177 00261	3D	186 00272	67	190 00276		

ENTRY POINTS TO SUBROUTINES NOT OUTPUT FROM LIBRARY.
Record CFILE

00000	RECORD	BCD	1	00071	22A	LDB	Y-56	00156	37A	LXO	22+8,1
00001	CSTHD	BCD	1	00072	FMP	3D+2		00157	38A	LDB	Y+1,1
00002	CFILD	BCD	1	00073	STO	Y-56		00160		STR	
00003	S	HTR		00074	23A	CAL	2D+2	00161	38A1	TXI	*+1,1,1
00004		HTR				B55		00162	38A2	TXL	38A,1,90
00005		HTR			00075	TSX	CSTHD,4			B55	
00006		BCD	1	00076	P2E	838		00163	40A	TSX	CFILD,4
00007		OUTPUT		00077	24A	LDB	W-69	00164	41A	CIA	II
00008		S20	5,1	00100	STR			00165	41A1	T2E	42A
00009		S20	5+1,2			LDB	V-1	00166		TPL	53A
00010		S20	S+2,4		00101			00167	42A	CRL	2D+3
00011	11A	B55		00102				00170		TSX	CSTHD,4
00012		TSX	PC0000,4	00103		LDB	V-2			P2E	82B
00013	12A	CLA	Y-69	00104		STR		00171		LXO	22+4,1
00014	12A1	T2E	139	00105		LDB	TE3	00172	43A	LDB	RECORD+1,1
00015		TPL	146	00106		STR		00173	44A	LDB	RECORD+1,1
00016	13A	CLA	22	00107		LDB	TE1	00174		STR	
00017		STO	11	00110		STR		00175	44A1	TXI	*+1,1,1
00018		CLA	Y-69	00111		LDB	TE2	00176	44A2	TXL	44A,1,12
00019	14A	FSD	3D	00112		STR		00177	45A	TSX	CFILD,4
00020		FSD	3D	00113		LDB	Y-6	00200	47A	CRL	2D+2
00021		FSD	3D	00114		STR		00201		TSX	CSTHD,4
00022		STO	Y-69	00115		LDB	V-7	00202		P2E	
00023	15A	CLA	Y-69	00116		STR		00203		LDB	22+4,1
00024		UPR	62					00204		TSX	RECORD+1,1
00025		LBS						00205		STR	
00026		UPR	62+1					00206	49A1	TXI	*+1,1,1
00027		LBS						00207	49A2	TXL	49A,1,12
00028		BLS	18					00208		B55	
00029		STO	1H					00210		TSX	CFILD,4
00030		STO	1H					00211		CRL	2D+3
00031		STO	1H					00212		TSX	CSTHD,4
00032	16A	LDB	Y-1	00121		TSX	CSTHD,4			B55	
00033		FPP	Y-1	00122		P2E	811			TSX	CSTHD,4
00034		STO	1D+1			B55				P2E	82C
00035		LDB	Y		00123					B55	
00036		FPP	Y	00124	26A	CAL	2D+2			TSX	CSTHD,4
00037		FPP	Y	00125		B55				CRL	2D+3
00038		FPP	Y-1	00126		P2E	828			TSX	CSTHD,4
00039		STO	1D+2	00127	27A	LDB	2D+6,1			B55	
00040		FPP	Y-1	00128	28A	LDB	Y+1,1			TSX	CSTHD,4
00041		LDB	Y-1	00129		STR				P2E	82C
00042		FPP	3D+1	00130				00213		B55	
00043		X55		00131						TSX	CSTHD,4
00044		FPP	Y	00132				00214		CRL	2D+3
00045		FPP	1D+2	00133	28A1	TXI	*+1,1,1			TSX	CSTHD,4
00046		STO	Y-38	00134	28A2	J2	28A,1,21			P2E	82C
00047	17A	LDB	Y+4	00135	30A	LDB	2D+4,1	00215		B55	
00048		FPP	3D+2	00136		LDB	Y+1,1	00217		STO	11
00049		STO	TE1	00137		STR		00220		TSX	CSTHD,4
00050		STO	TE1	00138						CRL	2D+3
00051		STO	TE1	00139						TSX	CSTHD,4
00052	18A	LDB	Y-5	00140	31A1	TXI	*+L,1,1	00221		B55	
00053		FPP	3D+2	00141	31A2	T2E	31A,1,11	00222		P2E	82C
00054		STO	TE1	00142		STR		00223		B55	
00055	19A	CLA	Y-3	00143		LDB	N-21	00224		STR	M-59
00056		FPP	Y-10	00144				00225		LDB	Y-1
00057		FPP	Y-34	00145				00227		STR	
00058		STO	Y-35	00146				00230		LDB	Y-82
00059		STO	Y-35	00147				00231		STR	
00060		STO	Y-16	00150				00232		LDB	Y-81
00061		STO	Y-16	00151	34A	LDB	2D+7,1	00233		STR	
00062		FPP	Y-36	00152	35A	LDB	Y+1,1	00234		LDB	Y-61
00063		FPP	Y-36	00153		STR		00235		STR	
00064		X55		00154	35A1	TXI	*+1,1,1	00236		LDB	Y-60
00065		FPP	3D+2	00155	35A2	TXI	35A,1,70	00237		TSX	
00066		STO	Y-37	00156							

RM 61 TMP-32

00240	STR	BCD 4-60		00331	BCD 1 IN. 5X
00241	LDR	BCD 4-60		00332	BCD 1 C5H0
00242	STR	BCD 4-60		00333	BCD 1 C5H0
00243	LDR	BCD 4-79		00334	BCD 1 3X. 128
00244	STR	BCD 4-79		00335	BCD 1 C1H1, 2
00245	BSS	BCD 4-79		00336	BCD 1 C1H1, 2
00246	TSX	CP11C. 4		00337	BCD 1 14. 72
00247	CLA	BCD 11		00340	BCD 1 1P8E
00248	SUB	BCD 22-5		00341	BCD 1 C1H1.
00249	T2E	BCD 57A		00342	BCD 1 8C5E
00250	T2E	BCD 57A		00343	BCD 1 C1H1.
00251	TPL	BCD 57A		00344	BCD 1 C1H1.
00252	TPL	BCD 56A		00345	BCD 1 C1H1.
00253	CLA	BCD 22		00346	BCD 1 C1H1.
00254	STO	BCD 11		00347	BCD 1 1P8E
00255	LWD	BCD 5. 1		00348	BCD 1 000000000000
00256	LWD	BCD 5. 1		00349	BCD 1 000000000000
00257	LWD	BCD 5. 2		00350	BCD 1 000000000000
00258	LWD	BCD 5. 2		00351	BCD 1 000000000000
00259	STO	BCD 1. 4		00352	BCD 1 000000000000
00260	TBL	BCD 1. 4		00353	BCD 1 000000000000
00261	BCD	BCD 1. 4		00354	BCD 1 000000000000
00262	OCT	BCD 00000020000000		00355	BCD 1 000000000000
00263	OCT	BCD 000006000000		00356	BCD 1 00000120000000
00264	OCT	BCD 000001000000		00357	BCD 1 000001000000
00265	OCT	BCD 000006200000		00358	BCD 1 000001100000
00266	OCT	BCD 000006200000		00359	BCD 1 00000670000000
00267	OCT	BCD 000001000000		00360	BCD 1 00000120000000
00268	OCT	BCD 000006200000		00361	BCD 1 000001000000
00269	OCT	BCD 000001000000		00362	BCD 1 114. 72
00270	OCT	BCD 000006200000		00363	BCD 1 C1H1.
00271	OCT	BCD 000001000000		00364	BCD 1 C1H1.
00272	OCT	BCD 000001000000		00365	BCD 1 C1H1.
00273	OCT	BCD 000001000000		00366	BCD 1 LT-02
00274	OCT	BCD 000001000000		00367	BCD 1 X. 6H0E
00275	OCT	BCD 000001000000		00368	BCD 1 X. 6H0E
00276	OCT	BCD 000001000000		00369	BCD 1 X. 6H0E
00277	OCT	BCD 000000077777		00370	BCD 1 17HHE
00278	OCT	BCD 000000000000		00371	BCD 1 10. 6X.
00279	OCT	BCD 000000000000		00372	BCD 1 10. 6X.
00280	OCT	BCD 000000000000		00373	BCD 1 111HHC1
00281	OCT	BCD 000000000000		00374	BCD 1 X.
00282	OCT	BCD 000000000000		00375	BCD 1 12. 6X.
00283	OCT	BCD 000000000000		00376	BCD 1 X. 6H0E
00284	OCT	BCD 000000000000		00377	BCD 1 X. 6H0E
00285	OCT	BCD 000000000000		00378	BCD 1 1HTR. 6
00286	OCT	BCD 000000000000		00379	BCD 1 1. 6H5L9

P. INPUT-OUTPUT

Table 4 illustrates the format that is necessary for the input data. Tables 5 and 6 illustrate the format of the output data that is currently obtained by use of Subroutines OUTONE and OUTPUT. These two subroutines can be easily modified. As they are presented in this report their primary purpose was to collect "debugging" data.

* DATA SIMPLE SPHERE 10**33/R**12 Z = 0.0 Y VERY SMALL FEB. 31961 E=26DG

10	66
1+1.000000E+00	
2+0.000000E+00	
3 6.200000E-09	
13 0.000000E+00	
14+2.640000E+02	
-15 9.000002E-01	
16 4.130000E-01	
17 2.600000E-01	
18 0.000000E+00	
19 6.39669112E-03	
20 1.000000E+00	
21 4.450000E-01	
22+2.619000E+02	
23 3.000000E-02	
24 1.9.000000E-01	
25 9.4577431E-01	
26 0.000000E+00	
27 0.000000E+00	
28 1.000000E+00	
29 1.000000E+00	
30 1.000000E+00	
31 0.000000E+00	
32 0.000000E+00	
33 0.000000E+00	
34 1.000000E+00	
35 1.000000E+00	
36 0.000000E+00	
37 0.000000E+00	
38 0.000000E+00	
39 0.000000E+00	
40 1.000000E+00	
41 0.000000E+00	
42 1.000000E+00	
43 9.000000E-01	
44 1.000000E+00	
45 0.000000E+00	
46 0.000000E+00	
47 5.000000E-01	
48 1.000000E+00	
49 1.000000E+00	
50 1.000000E+00	
51 0.000000E+00	
52 0.000000E+00	
53 4.000000E-01	
54 1.000000E+00	
55 1.000000E+00	
56 1.000000E+00	
57 0.000000E+00	
58 0.000000E+00	
59 1.000000E-04	
60 1.000000E+00	
61 0.000000E+00	
62 0.000000E+00	
63 0.000000E+00	
64 0.000000E+00	
65 1.000000E+00	
66 1.000000E+00	
67 0.000000E+00	
68 0.000000E+00	
69 1.000000E+00	
70 1.000000E+00	
71 0.000000E+00	
72 0.000000E+00	
73 0.000000E+00	
74 0.000000E+00	
75 0.000000E+00	
76 0.000000E+00	
-11 0.000000E+00	
SIMPL SPHERE 10**33/P**12 Z = 0.0 Y VERY SMALL FEB. 31961 E=33DG	
100 66	
17 9.300000E-01	
-21 0.000000E+00	
SIMPL SPHERE 10**33/Q**12 Z = 0.0 Y VERY SMALL FEB. 31961 E=36DG	
100 66	
17 9.400000E-01	
-21 0.000000E+00	
SIMPL SPHERE 10**33/R**12 Z = 0.0 Y VERY SMALL FEB. 31961 E=37DG	
100 66	
17 9.700000E-01	
-21 0.000000E+00	

Table 4. Input Data for a Spherical Ionosphere.

ORDINARY RAY 101SIMPLE SPHERE 10.*33/R**12 Z = 0.0 V VERY SMALL FEB. 3, 1961 E=260G

1.000000E 00 0.	6.280000E 09 0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	4.6059238E 00	1.570793E 00	7.2082097E-01
+.5373560E-13 0.	6.3563112E 03 1.0000000E 00	7.9412480E-11	4.6059238E 00	3.0000000E 02	1.0000000E 00	0.
5.4677431E 01 3.	0.	0.	0.	6.6569111E 03-4	9.581701E 02-4	6.394721E 03
+1.7436445E 03 1.0000000E 33 1.0000000E 01 0.	0.	0.	0.	0.	1.2000000E 03	0.
0.	-5.999998E-06 5.0000000E 01 1.0000000E 00 0.	0.	0.	5.0000000E-01 0.	0.	0.
1.000000E 00 0.	0.	-5.0743736E 02-4	7.486769E 03 4.1955717E 03 0.	0.	0.	0.
0.	0.	0.	0.	0.	3.0000000E 00 0.	0.
0.	1.0000000E 00-1 1.0000000E 00 3.9999993E-05 0.	0.	0.	0.	2.2802051E 00	0.
1.0000000E 00 2.3573103E 03 8.4965302E-01 4.6059233E 00	4.3866076E-01-3	9.865271E-01-2	34.34196E-06 2.2802051E 00	0.	0.	0.
1.0000000E 00 3.37E-06 0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.
1.0000000E 29 3.2302051E 00 1.0000000E 00 1.4383545E 00	4.8673434E 01 2.6189999E 02 4.3278773E-01-8	9.8559070E-01	0.	0.	0.	0.
-2.3435490E-03 3.2802051E 00 1.0000000E 00 1.4383545E 00	4.3878779E-01-1	4.132452E-04-4	9.081066E-12	1.2639290E-04	0.	0.
-5.2011474E-05 1.2951176E-12 1.0000000E 00 3.3333333E-06 0.	9.999998E-01-0.	9.999998E-01-0.	6.2800000E 09	0.	0.	0.
-5.9437342E-12 -2.4939416E-12 8.277476E-28 5.5619962E-13 5.0024484E-09 1.3281570E-15-5	5.1296621E-19	1.3516441E-20	0.	0.	0.	0.
5.6601182E 02-3.632445E-01 5.7858963E 04 1.0510761E 00 3.2802163E 00 2.2964180E-03 2.599452E 01 5.5473943E-03	0.	0.	0.	0.	0.	0.
-6.000000E 00-3.2802163E 00 2.599452E 01 2.0000000E 00 0.	0.	0.	0.	1.3267032E-08	5.4883252E-11	0.
1.0000000E 00-1.0000000E 00 9.96399919E-05 1.0000000E 00 0.	5.13556753E-03-1	1.1175871E-05-2	2.964180E-03	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.

Table 5-1. Output Data for "Debugging" Purposes from Tape 6.

RM 61 TMP-32

SIMPLE SPHERE 1D**33/R**12 Z = 0.0 V VERY SMALL FEB. 3, 1961 E=2603									
2.000000E 00	4.1802051E 00	1.000000E 00	1.8771973E 00	4.8665335E 01	2.638999E 02	4.3891473E-01	-8.9852868E-01		
-2.3436785E-03	4.2802051E 00	1.4267350E-05	0.	4.3891478E-01	-1.4130502E-04	-4.9086421E-12	1.2696661E-04		
6.2020958E-05	1.2952606E-12	1.000000E 00	3.3333333E-06	0.	6.9999998E-01	0.	6.280000E 09		
-5.1931012E-19	8.4201920E-28	-4.6710138E-13	5.1192869E-19	-1.3808698E-19	1.3808698E-20				
5.5901442E-02	5.6334660E-01	5.8481141E-04	1.0738028E-00	4.2801423E-00	2.9190779E-03	2.5996310E-01	3.689528E-03		
-0.	4.2801423E 00	0.	2.5995629E 01	3.0000000E 00	1.3265523E-06	8.6718626E-11			
1.2000000E 00	-1.0000000E 00	9.9999999E-05	2.0000030E 00	0.	4.3705307E-03	6.2823293E-05	-2.9190779E-03		
0.	0.	0.	0.	0.	0.	0.	0.	0.	
3.000000E 00	5.2302051E 00	1.000000E 00	2.3161621E 00	4.8657241E 01	2.6389999E 02	4.3904172E-01	-8.9846666E-01		
-2.3435080E-03	5.2802051E 00	1.7600684E-05	0.	4.3904173E-01	-1.4128551E-04	-4.9091517E-12	1.2694032E-04		
6.203233E-05	-1.2954039E-12	1.000000E 00	3.3333333E-06	0.	9.9999998E-01	0.	6.280000E 09		
-5.2695952E-19	8.3747120E-28	-6.7829534E-13	5.2395857E-19	-1.420931E-15	5.8643903E-19	1.4107853E-20			
5.5801687E 02	5.6345218E-01	5.9111220E 04	1.0970659E 00	5.2802444E 00	3.7182570E-03	2.5996522E 01	3.4701630E-03		
-0.	5.2802444E 00	0.	2.5996639E 01	3.0000000E 00	0.	1.3263984E-05	8.659731E-11		
1.000000E 00	-1.000000E 00	9.9999999E-05	3.0000000E 00	0.	3.3604530E-03	3.9637089E-05	-3.7182570E-03		
0.	0.	0.	0.	0.	0.	0.	0.	0.	
*.000000E 00	6.2602051E 00	1.000000E 00	2.7592490E 00	4.8649147E 01	2.6389999E 02	4.3916864E-01	-8.9840461E-01		
-2.3433371E-08	6.2802051E 00	2.0234017E-05	0.	4.3916866E-01	-1.4126600E-04	-4.9097344E-12	1.2691402E-04		
6.2039528E-05	-1.2955476E-12	1.000000E 00	3.3333333E-06	0.	9.9999998E-01	0.	6.280000E 09		
-5.3523936E-19	8.7141767E-28	-4.8977747E-13	5.3618962E-19	-1.3206091E-15	5.9910571E-19	1.4414011E-20			
5.5701944E-02	5.6355817E-01	5.9745170E 04	1.1206736E 00	6.2802376E 00	6.4677635E-03	2.5996768E 01	3.2315332E-03		
-0.	5.2802376E 00	0.	2.5996760E 01	3.0000000E 00	0.	1.3262444E-08	9.0519986E-11		
1.000000E 00	-1.000000E 00	9.3599999E-05	4.0000000E 00	0.	3.2392172E-03	3.2484531E-05	-4.4077635E-03		
0.	0.	0.	0.	0.	0.	0.	0.	0.	
5.000000E 00	7.2802051E 00	1.000000E 00	3.1945190E 00	4.86441053E 01	2.6389999E 02	4.3929555E-01	-8.9834257E-01		
-2.34406671E-03	7.2302051E 00	2.4267350E-05	0.	4.3929556E-01	-1.4124649E-04	-4.9102771E-12	1.2688773E-04		
6.2043535E-05	-1.2956098E-12	1.000000E 00	3.3333333E-06	0.	9.9999998E-01	0.	6.280000E 09		
-5.4634790E-19	8.6379793E-28	-6.8632668E-13	5.4878119E-19	-3.8703605E-15	6.1206927E-19	1.427737E-20			
5.5602198E-02	5.6365435E-01	6.0995159E 04	1.1452417E 00	7.2802491E 00	5.1159263E-03	2.5997538E 01	2.4420060E-03		
-0.	7.2802491E 00	0.	2.5998136E 01	3.0000000E 00	0.	3.2669032E-08	9.2687912E-11		
1.000000E 00	-1.000000E 00	9.9999999E-05	5.0000000E 00	0.	1.8640013E-03	4.3926623E-05	-5.159263E-03		
0.	0.	0.	0.	0.	0.	0.	0.	0.	
6.000000E 00	8.2802051E 00	2.000000E 00	3.6333350E 00	4.86329961E 01	2.6389999E 02	4.3942242E-01	-8.9828051E-01		
-2.34419267E-08	8.2802051E 00	2.7600683E-05	0.	4.3942243E-01	-1.4122699E-04	-4.9108201E-12	1.2686144E-04		
6.2052899E-05	-1.2958342E-12	1.000000E 00	3.3333333E-06	0.	9.9999998E-01	0.	6.280000E 09		
-5.6172867E-19	8.7479741E-28	-5.1363971E-13	5.6169183E-19	-4.0111253E-15	6.2533763E-19	1.5048139E-20			
5.5502456E-02	5.6377108E-01	6.1049324E 04	1.1701852E 00	8.2802662E 00	5.82965919E-03	2.5997531E 01	2.469134E-03		
-0.	8.2802662E 00	0.	2.5997436E 01	3.0000000E 00	0.	1.3259361E-08	9.4502312E-11		
1.000000E 00	-1.000000E 00	9.9999999E-05	6.0000000E 00	0.	2.5632419E-03	6.1039136E-05	-5.8296319E-03		
0.	0.	0.	0.	0.	0.	0.	0.	0.	
7.000000E 00	9.2802051E 01	2.0000000E 00	4.5129395E 00	4.8616781E 01	2.6389999E 02	4.3956760E-01	-8.9815238E-01		
-2.34445559E-08	1.02802051E 01	3.4267350E-05	0.	4.3967610E-01	-1.4118795E-04	-4.9119061E-12	1.2686885E-04		
6.20732645E-05	-1.2964069E-12	1.000000E 00	3.3333333E-06	0.	9.9999998E-01	0.	6.280000E 09		
-5.86331098E-19	8.7672796E-28	-9.3355968E-13	5.3875341E-19	-4.0280153E-15	6.5281622E-19	1.5712333E-20			
5.5302984E-02	5.6398525E-01	6.2382462E 04	1.2218503E 00	8.2802662E 00	5.82965919E-03	2.5998387E 01	1.6123513E-03		
-0.	1.0280153E 01	0.	2.5998160E 01	3.0000000E 00	0.	1.3256276E-08	9.867470E-11		
1.000000E 00	-1.000000E 00	9.9999999E-05	7.0000000E 00	0.	1.83945535E-03	5.2571297E-05	-7.1094036E-03		
0.	0.	0.	0.	0.	0.	0.	0.	0.	
8.000000E 00	9.2802051E 01	2.28802051E 00	5.3925781E 00	4.8606603E 01	2.6389999E 02	4.3992965E-01	-8.9803221E-01		
-2.34471515E-08	1.22802051E 01	4.0934017E-05	0.	4.3992966E-01	-1.4114891E-04	-4.9129924E-12	1.2675626E-04		
6.20959508E-05	-1.2964069E-12	1.000000E 00	3.3333333E-06	0.	9.9999998E-01	0.	6.280000E 09		
-5.861219902E-19	8.7672796E-28	-9.3355968E-13	5.3875341E-19	-4.0280153E-15	6.5281622E-19	1.5712333E-20			
5.5103992E-02	5.6420140E-01	6.3749802E 04	1.2759997E 00	8.2802662E 00	5.82965919E-03	2.5998387E 01	1.4576050E-03		
-0.	1.2280224E 01	0.	2.5998738E 01	3.0000000E 00	0.	1.3253187E-08	9.804772E-10		

Table 5-2. Output Data for "Debugging" Purposes from Tape 6.

SIMPLE SPHERE 10**33/R**12 Z = 0.0 V VERY SMALL FEB. 3, 1961 E22603

TH	SCANTR	V(22)	V(11)*C-B(C60)	V(10)-W(C60)	ANGLE E	DELTE	ANGLE A	DELTA-A
1	3.2802163E 00	3.2802051E 00	0.0-2.29641805E-03-1.	1.1758715E-05	2.5399864E 01	5.1358753E-03	0.	0.
2	4.2801423E 00	4.2802051E 00	0.0-2.91907795E-03-6.	2.823296E-05	2.5399529E 01	4.3706807E-03	0.	0.
3	5.2802443E 00	5.2802051E 00	0.-3.71825705E-03-3.	9.637089E-05	2.53996639E 01	3.3604530E-03	0.	0.
4	6.2802376E 00	6.2802051E 00	0.-4.4077635E-03-3.	2.484531E-05	2.53996160E 01	3.2392172E-03	0.	0.
5	7.2802491E 00	7.2802051E 00	0.-5.1159263E-03-4.	3.9286236E-05	2.5399836E 01	1.8640013E-03	0.	0.
6	8.2802652E 00	8.2802051E 00	0.-5.82965795E-03-6.	1.0351356E-05	2.5399736E 01	2.56322419E-03	0.	0.
7	1.0280153E 01	1.0280205E 01	0.-7.1094036E-03-5.	5.2571297E-05	2.53998160E 01	1.8394553E-03	0.	0.
8	1.2280224E 01	1.2280205E 01	0.-8.5737705E-03-1.	3.477440E-05	2.53998738E 01	1.2614505E-03	0.	0.
9	1.4280193E 01	1.4280205E 01	0.-9.9369287E-03-1.	1.801720E-05	2.53998787E 01	1.2123586E-03	0.	0.
10	1.6280214E 01	1.6280205E 01	0.-1.1350632E-02-8.	5.830698E-06	2.53998985E 01	1.0138562E-03	0.	0.
11	1.8280212E 01	1.8280205E 01	0.-1.2772560E-02-3.	7.193298E-06	2.53999053E 01	9.4662153E-04	0.	0.
12	2.0280135E 01	2.0280205E 01	0.-1.4060020E-02-6.	8.664551E-05	2.53999174E 01	8.2359971E-04	0.	0.
13	2.4280244E 01	2.4280205E 01	0.-1.6954660E-02-3.	3.8862228E-05	2.53999346E 01	6.5377716E-04	0.	0.
14	2.8280244E 01	2.8280205E 01	0.-1.9739389E-02-3.	6.93499345E 01	2.53999445E 01	6.55227127E-04	0.	0.
15	3.2280244E 01	3.2280205E 01	0.-2.25076638E-02-1.	3.5956449E-05	2.53999445E 01	5.5452598E-04	0.	0.
16	3.6280154E 01	3.6280205E 01	0.-2.5224486E-02-5.	5.034739E-05	2.53999454E 01	5.4556136E-04	0.	0.
17	4.0280238E 01	4.0280204E 01	0.-2.8095245E-02-3.	3.378601E-05	2.53999654E 01	3.4556488E-04	0.	0.
18	4.4280252E 01	4.4280205E 01	0.-3.0897141E-02-4.	8.6160553E-05	2.53999548E 01	4.5143281E-04	0.	0.
19	5.2280149E 01	5.2280204E 01	0.-3.6366940E-02-5.	5.7899498E-05	2.53999658E 01	3.4150946E-04	0.	0.
20	6.0280247E 01	6.0280204E 01	0.-4.2037487E-02-4.	1.48833E-05	2.53999701E 01	2.9903422E-04	0.	0.
21	6.8280211E 01	6.8280205E 01	0.-4.7575951E-02-6.	6.757202E-06	2.53999759E 01	2.4097761E-04	0.	0.
22	7.6280181E 01	7.6280205E 01	0.-5.3120613E-02-2.	2.6688184E-05	2.53999778E 01	2.2176770E-04	0.	0.
23	8.4280162E 01	8.4280205E 01	0.-5.8672905E-02-4.	2.9153445E-05	2.53999774E 01	2.2560968E-04	0.	0.
24	9.2280125E 01	9.2280204E 01	0.-6.210992E-02-7.	9.7353685E-05	2.53999810E 01	1.8932431E-04	0.	0.
25	1.0328013E 02	1.08280204E 01	0.-7.5426102E-02-1.	1.444028E-05	2.53999834E 01	1.6605897E-04	0.	0.
26	1.2428027E 02	1.2428020E 02	0.-8.6550498E-02-6.	6.757202E-05	2.53999837E 01	1.6243043E-04	0.	0.
27	1.4028022E 02	1.4028020E 02	0.-9.7742081E-02-1.	1.444092E-05	2.53999853E 01	1.468497E-04	0.	0.
28	1.5628027E 02	1.5628020E 02	0.-1.0649012E-01-6.	2.942505E-05	2.53999884E 01	1.1589977E-04	0.	0.
29	1.7226023E 02	1.7228020E 02	0.-1.2004662E-01-2.	4.79532E-05	2.53999872E 01	1.2742572E-04	0.	0.
30	1.8828029E 02	1.8828020E 02	0.-1.3125420E-01-8.	3.923340E-05	2.5399989E 01	1.107713E-04	0.	0.
31	2.2028024E 02	2.2028020E 02	0.-1.5339960E-01-3.	6.239624E-05	2.53999916E 01	6.4246466E-05	0.	0.
32	2.5228023E 02	2.5228020E 02	0.-2.1.7578316E-01-2.	6.702881E-05	2.53999925E 01	7.405193E-05	0.	0.
33	2.8428025E 02	2.8428020E 02	0.-2.1.9808560E-01-4.	1.961670E-05	2.53999924E 01	7.5985854E-05	0.	0.
34	3.1628023E 02	3.1628020E 02	0.-2.2.034973E-01-1.	1.444092E-05	2.53999950E 01	5.0199202E-05	0.	0.
35	3.4828026E 02	3.4828020E 02	0.-2.4.4254698E-01-1.	5.2538789E-05	2.53999937E 01	6.4240735E-03	0.	0.
36	3.9028060E 02	3.9028020E 02	0.-2.6.6443981E-01-4.	9.3591046E-05	2.53999938E 01	1.1461911E-04	0.	0.
37	4.1228375E 02	4.1228020E 02	0.-2.8361730E-01-3.	4.322275E-05	2.53999985E 01	1.1461911E-04	0.	0.
38	4.1228375E 02	4.1228020E 02	0.-2.8361730E-01-3.	4.322275E-05	2.53999985E 01	1.1461911E-04	0.	0.
39	4.2829058E 02	4.2828020E 02	0.-2.8804398E-01-2.	2.8888184E-05	2.53999919E 01	3.8163682E-04	0.	0.
40	4.431643E 02	4.431643E 02	0.-2.7328873E-01-3.	8.1466973E-06	2.53998635E 01	1.3647572E-03	0.	0.
41	4.6043275E 02	4.6043275E 02	0.-2.1.671496E-01-4.	5.776367E-05	2.53998735E 01	1.6240735E-03	0.	0.
42	4.1228375E 02	4.1228020E 02	0.-2.8361730E-01-3.	4.322275E-05	2.53999985E 01	1.1461911E-04	0.	0.
43	4.2028614E 02	4.2028020E 02	0.-2.8687668E-01-3.	3.1466973E-05	2.53999978E 01	1.2589964E-04	0.	0.
44	4.2829058E 02	4.2828020E 02	0.-2.8806305E-01-3.	3.057578E-05	2.539999618E 01	3.8163682E-04	0.	0.
45	4.3629912E 02	4.3629912E 02	0.-2.8503036E-01-3.	3.81466973E-05	2.53999311E 01	6.9878199E-04	0.	0.
46	4.431644E 02	4.431644E 02	0.-2.6.6443981E-01-4.	9.3591046E-05	2.53998628E 01	1.3711605E-03	0.	0.
47	4.6431645E 02	4.6431645E 02	0.-2.8361730E-01-3.	4.322275E-05	2.53999985E 01	1.1461911E-04	0.	0.
48	4.1831113E 02	4.1831113E 02	0.-2.6.126362E-01-3.	4.332275E-05	2.53998039E 01	1.9606912E-03	0.	0.
49	4.5239269E 02	4.5239269E 02	0.-2.4.221930E-01-1.	9.073498E-05	2.53997151E 01	2.884023E-03	0.	0.
50	4.5639447E 02	4.5639447E 02	0.-2.1.312332E-01-6.	8.664551E-05	2.53995794E 01	4.2054754E-03	0.	0.

Table 6-1. Output Data from Tape 10.

RM 61TMP-32

SIMPLE SPHERE 10**33/R**12 Z = 0.0 VERY SMALL FEB. 3, 1961 E=260G

IN	SLANTR	VC23	VC113*C-V60)	VC103-B(C60)	ANGLE E	ANGLE A	DELT-A
51	4.6043246E 02	4.9028020E	02-1.6741943E-01	6.4849854E-05	2.5923686E-01	6.3136561E-03	0.
52	4.6450579E 02	4.6428020E	02-9.5439911E-02	1.4495850E-04	2.5990346E-01	9.6538320E-03	0.
53	4.6450579E 02	4.6428020E	02-9.5439911E-02	1.4495850E-04	2.5990346E-01	9.6538320E-03	0.
54	4.6655664E 02	4.6628020E	02-4.477302E-02	2.2888184E-04	2.5987965E-01	1.2034366E-02	0.
55	4.6862007E 02	4.6862007E	02-1.980402E-02	4.2724607E-04	2.5984883E-01	1.3116489E-02	0.
56	4.7070059E 02	4.7028020E	02-1.0268902E-01	7.2479244E-04	2.5980896E-01	1.9103399E-02	0.
57	4.7280349E 02	4.7228020E	02-2.1018882E-01	1.1177063E-03	2.5975617E-01	2.4382921E-02	0.
58	4.7493609E 02	4.7428020E	02-3.5182371E-01	1.9416809E-03	2.5968559E-01	3.1440641E-02	0.
59	4.7710944E 02	4.7628020E	02-5.4186630E-01	3.4022710E-03	2.59588918E-01	4.1081667E-02	0.
60	4.7710944E 02	4.7628020E	02-5.4186630E-01	3.4022710E-03	2.59588918E-01	4.1081667E-02	0.
61	4.7821616E 02	4.7728020E	02-6.6185760E-01	4.6889262E-03	2.5952773E-01	4.7266182E-02	0.
62	4.7933971E 02	4.7828020E	02-8.0370112E-01	6.3629150E-03	2.5945338E-01	5.4661363E-02	0.
63	4.8043387E 02	4.7928020E	02-9.7331001E-01	8.7776184E-03	2.5936385E-01	6.3614247E-02	0.
64	4.8165315E 02	4.8128020E	02-1.1788902E-00	1.2409210E-02	2.5925401E-01	7.4598472E-02	0.
65	4.8285367E 02	4.8128020E	02-1.4322819E-00	1.8035989E-02	2.5911684E-01	8.8151413E-02	0.
66	4.8409356E 02	4.8228020E	02-1.7513924E-00	2.6840210E-02	2.5894148E-01	1.0585171E-01	0.
67	4.8409356E 02	4.8228020E	02-1.7513924E-00	2.6840210E-02	2.5894148E-01	1.0585171E-01	0.
68	4.8538543E 02	4.83278902E-02	1.9446346E-00	3.3222198E-02	2.5883437E-01	1.165273E-01	0.
69	4.8605252E 02	4.8378020E	02-2.1659506E-00	4.166412E-02	2.5871027E-01	1.2897275E-01	0.
70	4.8605252E 02	4.8378020E	02-2.1659506E-00	4.166412E-02	2.5871027E-01	1.2897275E-01	0.
71	4.8674425E 02	4.8428020E	02-2.72653320E	6.863403E-02	2.5639092E-01	1.60920730E-01	0.
72	4.8674425E 02	4.8428020E	02-2.72653320E	6.863403E-02	2.5639092E-01	1.60920730E-01	0.
73	4.875352E 02	4.8478020E	02-3.0903854E-00	9.3740201E-02	2.5818072E-01	1.8192742E-01	0.
74	4.881913E 02	4.85328020E	02-3.5360903E-00	1.2361516E-01	2.5792013E-01	2.0798624E-01	0.
75	4.8865359E 02	4.85798020E	02-4.0956358E-00	1.7445759E-01	2.5778884E-01	2.4111571E-01	0.
76	4.8973793E 02	4.8628020E	02-4.8191681E-00	2.585640E-01	2.5715396E-01	2.8460416E-01	0.
77	4.8973793E 02	4.8628020E	02-4.8191681E-00	2.585640E-01	2.5715396E-01	2.8460416E-01	0.
78	4.9013373E 02	4.8653020E	02-5.2637558E-00	3.2129969E-01	2.5688341E-01	3.1165854E-01	0.
79	4.9052556E 02	4.91679302E	02-5.7776580E-00	4.05245656E-01	2.5127195E-01	2.56220D41E-01	0.
80	4.9052556E 02	4.91679302E	02-6.3705139E-00	5.1927195E-01	2.56220D41E-01	3.7925803E-01	0.
81	4.9157535E 02	4.91753020E	02-7.8001518E-00	8.7360382E-01	2.5577706E-01	4.2229416E-01	0.
82	4.9157535E 02	4.91753020E	02-7.8001518E-00	8.7360382E-01	2.5577706E-01	4.2229416E-01	0.
83	4.9184041E 02	4.87778020E	02-8.6133842E-00	1.1283607E-00	2.5477955E-01	5.2204480E-01	0.
84	4.9184041E 02	4.87778020E	02-8.6133842E-00	1.1283607E-00	2.5477955E-01	5.2204480E-01	0.
85	4.9195053E 02	4.8790520E	02-9.0341530E-00	1.2757263E-00	2.5450830E-01	5.4917005E-01	0.
86	4.9232947E 02	4.8803020E	02-9.4594002E-00	1.4352884E-00	2.5423270E-01	5.7672285E-01	0.
87	4.9232947E 02	4.8815520E	02-9.8860130E-00	1.6060143E-00	2.5395513E-01	6.0448710E-01	0.
88	4.9237142E 02	4.8890520E	02-1.0310882E-01	1.7865007E-00	2.5367702E-01	6.3229750E-01	0.
89	4.9238144E 02	4.8903020E	02-1.0731931E-01	1.973461E-00	2.534001869E-01	6.5997450E-01	0.
90	4.9238144E 02	4.8915520E	02-1.147602E-01	2.1705627E-00	2.5116259E-01	6.8740838E-01	0.
91	4.9235436E 02	4.8878020E	02-1.1958562E-01	2.5279800E-00	2.5258763E-01	7.4123710E-01	0.
92	4.9237142E 02	4.8995520E	02-1.4954005E-01	2.3525257E-01	2.5323248E-01	7.6755168E-01	0.
93	4.9238144E 02	4.8903020E	02-1.2738235E-01	3.0001869E-00	2.5206582E-01	7.9341803E-01	0.
94	4.9238521E 02	4.8915520E	02-1.3116363E-01	3.219734E-00	2.5181148E-01	8.1885195E-01	0.
95	4.9238352E 02	4.8928020E	02-1.3486749E-01	3.4314194E-00	2.5156157E-01	8.4383259E-01	0.
96	4.9237704E 02	4.8940520E	02-1.3849567E-01	3.690135E-00	2.5131639E-01	8.6839045E-01	0.
97	4.9235157E 02	4.8965520E	02-1.4554005E-01	4.0866089E-00	2.5083744E-01	9.16257578E-01	0.
98	4.9231303E 02	4.8990520E	02-1.5231567E-01	4.524957E-00	2.503748E-01	9.6256191E-01	0.
99	4.9226383E 02	4.9015520E	02-1.5885216E-01	4.9630165E-00	2.4992456E-01	1.0074546E-00	0.
100	4.9220610E 02	4.9040520E	02-1.6517315E-01	5.3998146E-00	2.4948932E-01	1.0510677E-00	0.

SIMPLE SPHERE 10*33/R**12 Z = 0.0 V VEPY SMALL FEB. 3, 1961 E=260J

IN	ELANTR	V(2)	V(11)*C-W(C60)	J(10)-W(C60)	ANGLE E	ANGLE A	DELT-H
101	4.9214146E 02	4.9065520E 02	1.7130016E 01	5.3347353E 01	2.47676487E 01	1.6535124E 00	0.
102	4.9207153E 02	4.9095520E 02	1.7724288E 01	6.2670479E 01	2.4865943E 01	1.1349063E 00	0.
103	4.9199638E 02	4.9115520E 02	1.8204638E 01	6.3669757E 00	2.4821646E 01	1.1753520E 00	0.
104	4.9183705E 02	4.9165520E 02	1.9420552E 01	7.5566183E 00	2.476559E 01	1.2537403E 00	0.
105	4.9166635E 02	4.9215520E 02	2.0489605E 01	8.3897247E 00	2.4670725E 01	1.3292749E 00	0.
106	4.9158804E 02	4.9215520E 02	2.1518730E 01	9.204819E 00	2.4592750E 01	1.4024301E 00	0.
107	4.9130246E 02	4.9215520E 02	2.251390E 01	1.0041428E 01	2.4526442E 01	1.4735602E 00	0.
108	4.9111628E 02	4.92365520E 02	2.3480366E 01	1.0853451E 01	2.4457020E 01	1.542979E 00	0.
109	4.9092530E 02	4.9415520E 02	2.4421440E 01	1.1657089E 01	2.4389100E 01	1.6108993E 00	0.
110	4.9073188E 02	4.9465520E 02	2.5340874E 01	1.2453209E 01	2.4222460E 01	1.6775393E 00	0.
111	4.9053696E 02	4.9545520E 02	2.6240871E 01	1.2624081E 01	2.42562945E 01	1.7430547E 00	0.
112	4.9034063E 02	4.9565520E 02	2.7123058E 01	1.4024539E 01	2.4192421E 01	1.8075784E 00	0.
113	4.9014326E 02	4.9615520E 02	2.7991978E 01	1.4801239E 01	2.4128764E 01	1.871235E 00	0.
114	4.9994538E 02	4.9665520E 02	2.8846248E 01	1.5572197E 01	2.4065884E 01	1.9341151E 00	0.
115	4.9974698E 02	4.9715520E 02	2.9688234E 01	1.6338116E 01	2.4003682E 01	1.9963179E 00	0.
116	4.9954836E 02	4.9765520E 02	2.97651959E 01	1.7069239E 01	2.3942135E 01	2.0578862E 00	0.
117	4.9915053E 02	4.9865520E 02	3.2152359E 01	1.8608814E 01	2.3820574E 01	2.179425E 00	0.
118	4.9875238E 02	4.9965520E 02	3.3751873E 01	2.0103142E 01	2.3700806E 01	2.2991937E 00	0.
119	4.9835530E 02	5.0065520E 02	3.5723261E 01	2.1584251E 01	2.3582515E 01	2.4174844E 00	0.
120	4.9795963E 02	5.0165520E 02	3.6870712E 01	2.3053741E 01	2.3465406E 01	2.5345933E 00	0.
121	4.9756485E 02	5.0265520E 02	3.8397495E 01	2.4512874E 01	2.3435931E 01	2.6502886E 00	0.
122	4.9717145E 02	5.0365520E 02	3.9906414E 01	2.5962818E 01	2.3234048E 01	2.7659513E 00	0.
123	4.9677946E 02	5.0465520E 02	4.139989E 01	2.7404682E 01	2.3119443E 01	2.8805563E 00	0.
124	4.9638615E 02	5.0565520E 02	4.2879550E 01	2.8393085E 01	2.3005613E 01	2.9945872E 00	0.
125	4.9600050E 02	5.0665520E 02	4.4347145E 01	3.0666823E 01	2.2818454E 01	3.1081551E 00	0.
126	4.9561363E 02	5.0765520E 02	4.5803897E 01	3.1638398E 01	2.2776679E 01	3.2213202E 00	0.
127	4.9484513E 02	5.0965520E 02	4.6862544E 01	3.4515350E 01	2.2553197E 01	3.4468029E 00	0.
128	4.9408331E 02	5.1165520E 02	5.1543705E 01	3.7323277E 01	2.2328518E 01	3.6714813E 00	0.
129	4.9329776E 02	5.1365520E 02	5.4371769E 01	4.0114696E 01	2.2104323E 01	3.8956767E 00	0.
130	4.9325830E 02	5.1565520E 02	5.7177579E 01	4.5655311E 01	2.1656430E 01	4.3435633E 00	0.
131	4.9184375E 02	5.1765520E 02	5.996867E 01	5.1148151E 01	2.1208161E 01	4.7518387E 00	0.
132	4.9111172E 02	5.1965520E 02	6.2735428E 01	4.8407218E 01	2.1432406E 01	4.5675936E 00	0.
133	4.9038703E 02	5.2165520E 02	6.54919134E 01	5.1148151E 01	2.1208161E 01	4.7518387E 00	0.
134	4.9066989E 02	5.2365520E 02	6.8236904E 01	5.6532250E 01	2.0758705E 01	5.0163895E 00	0.
135	4.7996012E 02	5.2565520E 02	7.9964904E 01	6.0033375E 01	2.0533375E 01	5.4666247E 00	0.
136	4.7825773E 02	5.2765520E 02	7.3683174E 01	5.9331753E 01	2.0533375E 01	5.4666247E 00	0.
137	4.7697539E 02	5.3165520E 02	7.9090130E 01	6.4706783E 01	2.0533375E 01	5.4666247E 00	0.
138	4.7532298E 02	5.3565520E 02	8.4458717E 01	7.0068665E 01	1.9627151E 01	6.3722420E 00	0.
139	4.7420059E 02	5.3965520E 02	8.9791607E 01	7.5377016E 01	1.9170812E 01	6.8291877E 00	0.
140	4.7290850E 02	5.4365520E 02	9.3090392E 01	8.0694469E 01	1.8712172E 01	7.2878224E 00	0.
141	4.7164694E 02	5.4765520E 02	1.0035973E 02	8.5939381E 01	1.8251248E 01	7.7467514E 00	0.
142	4.7041619E 02	5.5165520E 02	1.0558635E 02	9.1192539E 01	1.7787991E 01	8.2120034E 00	0.
143	4.6221613E 02	5.5565520E 02	1.1078929E 02	9.6394352E 01	1.7322420E 01	8.6777301E 00	0.
144	4.6004739E 02	5.5965520E 02	1.1597570E 02	1.0156436E 02	1.6854547E 01	9.1454523E 00	0.
145	4.6691003E 02	5.6365520E 02	1.2109423E 02	1.0670266E 02	1.6364413E 01	9.6158864E 00	0.
146	4.6473008E 02	5.7165520E 02	1.3127106E 02	1.1680383E 02	1.5437478E 01	1.0562522E 01	0.
147	4.6267830E 02	5.7965520E 02	1.4131864E 02	1.2693630E 02	1.4461923E 01	1.1518062E 01	0.
148	4.6073640E 02	5.8765520E 02	1.5123579E 02	1.3685859E 02	1.3518227E 01	1.2481773E 01	0.
149	4.5996591E 02	5.9565520E 02	1.6102116E 02	1.4664932E 02	1.2546731E 01	1.3457255E 01	9.8411701E-03
150	4.5730842E 02	6.0365520E 02	1.7067335E 02	1.5670695E 02	1.1676782E 01	1.4431113E 01	0.

Table A-3 Output Data from Tape 10

RM 61TMP-32

SIMPLE SPHERE 10**33/R**412 Z = 0.0 V VERY SMALL FEB. 3, 1961 E=266J

IN	SLANT R	U(25)	U(133)-U(60)	U(133)-U(60)	ANGLE E	DELTA-E	ANGLE H	DELTA-H
1	151	4.5378519E-02	6.1165520E-02	1.8019119E-02	1.-6583038E-02	1.-0582244E-01	1.-5417756E-01	0.
1	152	4.5314751E-02	6.2765520E-02	1.9881790E-02	1.-8446804E-02	0.8.5925636E-00	0.1.7407436E-01	0.
1	153	4.5106232E-02	6.4365520E-02	2.1689203E-02	2.0255377E-02	0.6.5820499E-00	0.1.9417950E-01	0.
1	154	4.4953725E-02	6.5965520E-02	2.3440598E-02	2.2007836E-02	0.4.553746E-00	0.2.1444625E-01	0.
1	155	4.4857310E-02	6.7565520E-02	2.5135401E-02	2.3703752E-02	0.2.5174736E-00	0.2.3482526E-01	0.
1	156	4.4818835E-02	6.9165520E-02	2.6773260E-02	2.5342726E-02	0.4.7352738E-01	0.2.5526472E-01	0.
1	157	4.4836954E-02	7.0765520E-02	2.8354027E-02	2.6924607E-02	0.2.4428460E-00	0.2.4428460E-01	0.
1	158	4.5044008E-02	7.3965520E-02	3.1344744E-02	2.9917554E-02	0.2.5.6436183E-00	0.2.0356381E-01	0.
1	159	4.5475844E-02	7.7165520E-02	3.4110675E-02	3.2685715E-02	0.2.9.6583842E-00	0.1.6341610E-01	0.
1	160	4.5126162E-02	8.0365520E-02	3.6653133E-02	3.5235401E-02	0.1.3578833E-01	0.1.2421166E-01	0.
1	161	4.5985934E-02	8.3565520E-02	3.8996181E-02	3.7575679E-02	0.1.7373080E-01	0.8.6269197E-01	0.
1	162	4.5043732E-02	8.6765520E-02	4.1136019E-02	3.9717800E-02	0.2.2.1015699E-01	0.4.9843011E-01	0.
1	163	4.51427055E-02	8.9965520E-02	4.3090550E-02	4.1674507E-02	0.2.2.4468012E-01	0.1.5119870E-02	0.8.8911701E-03
1	164	4.5274979E-02	9.6365520E-02	4.6498168E-02	4.5036583E-02	0.2.3.0879896E-01	0.1.-4.8798959E-00	0.
1	165	4.5832539E-02	1.0276552E-02	4.9315192E-02	4.7922024E-02	0.2.3.6519111E-01	0.1.-1.0519111E-01	0.
1	166	4.58777833E-02	1.0916552E-02	5.1686395E-02	5.0283729E-02	0.2.4.1440417E-01	0.1.-1.5440418E-01	0.
1	167	4.6040655E-02	1.1565520E-02	5.3655111E-02	5.2269904E-02	0.2.4.571284E-01	0.1.-1.9.12850E-01	0.
1	168	4.6034130E-02	1.2165520E-02	5.5321150E-02	5.3921402E-02	0.2.4.9417751E-01	0.1.-2.3417752E-01	0.
1	169	4.6137939E-02	1.2836552E-02	5.8732103E-02	5.5332812E-02	0.2.5.2635685E-01	0.1.-2.6635687E-01	0.
1	170	4.6156156E-02	1.4116552E-02	6.8930389E-02	5.7600017E-02	0.2.5.7894338E-01	0.1.-3.1894338E-01	0.
1	171	4.6263752E-02	1.5396552E-02	6.0576265E-02	5.9304810E-02	0.2.6.191260E-01	0.1.-3.5961260E-01	0.
1	172	4.64171325E-02	1.6676552E-02	6.1991215E-02	6.0628677E-02	0.2.6.5170284E-01	0.1.-3.9170284E-01	0.
1	173	4.6487916E-02	1.7296552E-02	6.2935113E-02	6.1631892E-02	0.2.6.7751948E-01	0.1.-4.1751942E-01	0.
1	174	4.6561444E-02	1.8236552E-02	6.3562643E-02	6.2537339E-02	0.2.6.9865881E-01	0.1.-4.3865881E-01	0.

Table 6-4. Output Data from Tape 10

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